

Test #3
MATH 1634

April 17, 2026

DIRECTIONS: This is the third test for this section of MATH 1634. The test contains six problems counting various point values each for a total of 100 points. The value of each problem is indicated with the problem. You must complete all the problems. You must show all your work clearly and completely in the spaces provided. You may use your calculator, but you may not use any printed or online reference nor give assistance to or receive assistance from anyone. If you violate these rules, you will fail the course.

Good luck.

My signature below indicates that I have read and understand the instructions printed above and I agree to abide by them.

Name (printed): _____

Problem 1. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing? (20 pts)

Solution. Let ℓ be the length of the rectangle and w be the width of the rectangle. We are given that $d\ell/dt = 8$ and $dw/dt = 3$.

The area of the rectangle is

$$A = \ell w.$$

Taking the derivative with respect to t , we get

$$\frac{dA}{dt} = \frac{d\ell}{dt}w + \ell \frac{dw}{dt}.$$

The problem tells us that $\ell = 20$ and $w = 10$. Substituting these values and the value of the two derivatives and computing, we get

$$\frac{dA}{dt} = (8)(10) + (20)(3) = 140 \text{ cm}^2/\text{s}.$$

Problem 2. Find the absolute maximum and absolute minimum values of the function

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

on the interval $[-2, 3]$. (15 pts)

Solution. This is a continuous function on a closed interval, so we know it has both a maximum value and a minimum value and that these values must occur at either a critical point or an endpoint of the interval.

First, we compute the derivative and find the critical points.

$$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x - 2)(x + 1).$$

This derivative is defined everywhere and is zero if $x = -1$ or $x = 2$. Both these critical points are in the given interval.

We construct a T-chart and find the values of f at the critical points and the endpoints of the interval.

| | |
|-----|-----|
| x | y |
| -2 | -3 |
| -1 | 8 |
| 2 | -19 |
| 3 | -8 |

From this we see that the absolute maximum value of f on the interval $[-2, 3]$ is 8 and the absolute minimum value of f on the interval $[-2, 3]$ is -19.

Problem 3. (15 pts) Consider the function

$$f(x) = x^4 - 2x^2 + 3.$$

- a) Find the intervals on which f is increasing or decreasing.
- b) Find the local maximum and minimum values of f .
- c) Find the intervals of concavity and the inflection points.

Solution. a) To find the values on which the function is increasing or decreasing, we take the derivative of f and find out where it is positive or negative.

The derivative of f is $f'(x) = 4x^3 - 4x = 4x(x - 1)(x + 1)$, so the zeros of the first derivative are $x = -1$, $x = 0$, and $x = 1$. By either using test points or by graphing the derivative, we find that the derivative is positive on the intervals $(-1, 0)$ and $(1, \infty)$ and the derivative is negative on the intervals $(-\infty, -1)$ and $(0, 1)$.

Hence, f is increasing on the intervals $(-1, 0)$ and $(1, \infty)$ and f is decreasing on the intervals $(-\infty, -1)$ and $(0, 1)$.

- b) By the First Derivative Test, there is a local maximum at $x = 0$ and there is a local minima at $x = -1$ and $x = 1$. The local maximum value at $x = 0$ is $f(0) = 3$. The local minimum value at both $x = \pm 1$ is $f(\pm 1) = 2$.
- c) Taking the second derivative, we get $f''(x) = 12x^2 - 4 = 4(3x^2 - 1)$, and the zeros of the second derivative are $x = \pm\sqrt{3}/3$. By either using test points or by graphing the second derivative, we find that the second derivative is positive on the intervals $(-\infty, -\sqrt{3}/3)$ and $(\sqrt{3}/3, \infty)$ and the second derivative is negative on the interval $(-\sqrt{3}/3, \sqrt{3}/3)$.

Hence, f is concave up on the intervals $(-\infty, -\sqrt{3}/3)$ and $(\sqrt{3}/3, \infty)$ and f is concave down on the interval $(-\sqrt{3}/3, \sqrt{3}/3)$. The two points $(\pm\sqrt{3}/3, 22/9)$ are inflection points.

Problem 4. Use L'Hôpital's Rule to evaluate the following limits: (10 pts each)

$$(a) \lim_{x \rightarrow \pi/2} \frac{1 - \sin x}{1 + \cos 2x}$$

Solution. This limit looks like $0/0$, so we apply L'Hôpital's Rule (twice):

$$\begin{aligned} \lim_{x \rightarrow \pi/2} \frac{1 - \sin x}{1 + \cos 2x} &= \lim_{x \rightarrow \pi/2} \frac{-\cos x}{-2 \sin 2x} \\ &= \lim_{x \rightarrow \pi/2} \frac{\sin x}{-4 \cos 2x} \\ &= \frac{\sin(\pi/2)}{-4 \cos \pi} \\ &= \frac{1}{4}. \end{aligned}$$

$$(b) \lim_{x \rightarrow 0} (\csc x - \cot x)$$

Solution. This limit looks like $\infty - \infty$, so we do a little algebraic manipulation.

$$\csc x - \cot x = \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \frac{1 - \cos x}{\sin x}.$$

Now, the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x}$$

looks like $0/0$, so we apply L'Hôpital's Rule :

$$\begin{aligned} \lim_{x \rightarrow 0} (\csc x - \cot x) &= \lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x} \\ &= \lim_{x \rightarrow 0} \frac{\sin x}{\cos x} \\ &= \frac{\sin 0}{\cos 0} \\ &= 0. \end{aligned}$$

Problem 5. (20 pts) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.

Solution. The distance between the point (x, y) and the point $(1, 0)$ is

$$d = \sqrt{(x-1)^2 + y^2}.$$

We need to maximize this function. If we maximize the square of the function, we have maximized the function, so we maximize

$$(x-1)^2 + y^2 = x^2 - 2x + 1 + y^2.$$

This has too many variables, so we use the relation $4x^2 + y^2 = 4$ to eliminate y from the problem:

$$g(x) = x^2 - 2x + 1 + (4 - 4x^2) = 5 - 2x - 3x^2.$$

This is a parabola opening downward and the maximum occurs at the vertex—which is where the derivative is zero. The derivative is $g'(x) = -6x - 2$ and this is zero at $x = -1/3$.

So, the points on the ellipse farthest from the point $(1, 0)$ are the points $\left(-\frac{1}{3}, \pm \frac{4\sqrt{2}}{3}\right)$.

Problem 6. (10 pts) Find the maximum value of the function

$$f(x) = 3 \cos^2 x + 2 \sin x + 1.$$

HINT: Since the function is periodic, you can look at the interval $[0, 2\pi]$. Make sure you find ALL the critical points in this interval.

Solution. Since $\sin x$ and $\cos x$ are periodic of period 2π , f is also periodic of period 2π . So, we can look for a maximum value of f on the interval $[0, 2\pi]$.

We find the critical points of the function f :

$$\begin{aligned} f'(x) &= -6 \sin x \cos x + 2 \cos x \\ &= -2 \cos x(3 \sin x - 1). \end{aligned}$$

The derivative is zero where $\cos x = 0$ and where $\sin x = \frac{1}{3}$.

For $\cos x = 0$, there are two critical points in the interval $[0, 2\pi]$: $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

For $\sin x = \frac{1}{3}$, there are two critical points in the interval $[0, 2\pi]$: $x = \arcsin\left(\frac{1}{3}\right)$ and $x = \pi - \arcsin\left(\frac{1}{3}\right)$.

Now we put these on a T-chart along with the endpoints and evaluate:

| x | $f(x)$ |
|---|----------------|
| 0 | 4 |
| $\arcsin\left(\frac{1}{3}\right)$ | $\frac{13}{3}$ |
| $\frac{\pi}{2}$ | 3 |
| $\pi - \arcsin\left(\frac{1}{3}\right)$ | $\frac{13}{3}$ |
| $\frac{3\pi}{2}$ | -1 |
| 2π | 4 |

The maximum value of f is then $\frac{13}{3}$.