Final Examination
MATH 1634
Monday, May 11, 2026

Directions: *This is the final examination for this section of MATH 1634. The examination contains twenty problems counting various point values each for a total of 250 points. The value of each problem is indicated at the beginning of each section of the examination. You may use a calculator but you may* **not** *use a calculator which does symbolic differentiation or integration, such as the TI-89 or TI-92. You may not use any reference, online or physical, and you may not give assistance to or receive assistance from anyone. To do so will result in your receiving a grade of F for the course. All your work must be shown clearly and legibly in the space provided.*

Good luck.

My signature below indicates that I have read and understand the instructions printed above and I agree to abide by them.

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Name (printed):______________________________

Problem 1. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why. (12 pts)

(a) $\lim_{x\to 2^-} f(x)$ (b) $\lim_{x\to 2^+} f(x)$ (c) $\lim_{x\to 2} f(x)$

(d) $f(2)$ (e) $\lim_{x\to 4} f(x)$ (f) $f(4)$

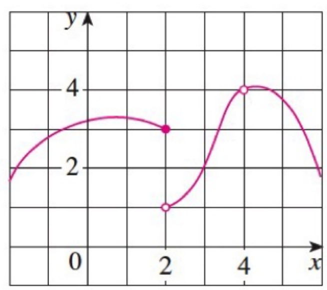

Figure 1: Figure for Problem 1.

Answers:

(a) ____ 3 ____

(b) ____ 1 ____

(c) Does not exist since the two one-sided limits aren't equal.

(d) ____ 3 ____

(e) ____ 4 ____

(f) $f(4)$ is undefined.

Problem 2. Refer to Figure 1 for Problem 1. (15 pts)

(a) Is the function f continuous at $x = 0$? Why or why not?

Solution. Yes, f is continuous at $x = 0$ because $\lim_{x \to 0} f(x) = f(0)$.

(b) Is the function f continuous at $x = 2$? Why or why not?

Solution. No, f is not continuous at $x = 2$ because $\lim_{x \to 2} f(x)$ does not exist.

(c) Is the function f continuous at $x = 4$? Why or why not?

Solution. No, f is not continuous at $x = 4$ because $f(4)$ is undefined.

Problem 3. (10 pts) Evaluate the limit

$$\lim_{t \to -2} \frac{t^4 - 2}{2t^2 - 3t + 2}.$$

Solution.

$$\begin{aligned}\lim_{t \to -2} \frac{t^4 - 2}{2t^2 - 3t + 2} &= \frac{(-2)^4 - 2}{2(-2)^2 - 3(-2) + 2} \\ &= \frac{7}{8}.\end{aligned}$$

Problem 4. (10 pts) Evaluate the limit

$$\lim_{x \to 5} \frac{x^2 - 6x + 5}{x - 5}.$$

Solution.

$$\begin{aligned}\lim_{x \to 5} \frac{x^2 - 6x + 5}{x - 5} &= \lim_{x \to 5} \frac{(x-5)(x-1)}{x - 5} \\ &= \lim_{x \to 5} (x - 1) \\ &= 5 - 1 \\ &= 4.\end{aligned}$$

Problem 5. (15 pts) Evaluate the limit

$$\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right).$$

Solution. First,

$$-1 \leq \cos\left(\frac{1}{x}\right) \leq 1$$

for all x, so

$$-x^2 \leq x^2 \cos\left(\frac{1}{x}\right) \leq x^2.$$

Since $\lim_{x \to 0} \pm x^2 = 0$, $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right) = 0$, as well, by the Squeeze Theorem.

Problem 6. (10 pts) Find the derivative of $f(x) = x^5 + 4x^3 + 2x + 1$.

Solution.

$$f'(x) = 5x^4 + 12x^2 + 2.$$

Problem 7. (10 pts) Find the derivative of $f(x) = xe^x$.

Solution.

$$\begin{aligned} f'(x) &= (xe^x)' \\ &= (x)'e^x + x(e^x)' \\ &= (1)e^x + xe^x \\ &= e^x + xe^x \\ &= e^x(1 + x). \end{aligned}$$

Problem 8. (10 pts) Find the derivative of $f(x) = \dfrac{x^2 + x - 2}{x^3 + 6}$.

Solution.

$$\begin{aligned}
f'(x) &= \left(\frac{x^2 + x - 2}{x^3 + 6}\right)' \\
&= \frac{(x^2 + x - 2)'(x^3 + 6) - (x^2 + x - 2)(x^3 + 6)'}{(x^3 + 6)^2} \\
&= \frac{(2x + 1)(x^3 + 6) - (x^2 + x - 2)(3x^2)}{(x^3 + 6)^2} \\
&= \frac{(2x^4 + x^3 + 12x + 6) - (3x^4 + 3x^3 - 6x^2)}{(x^3 + 6)^2} \\
&= \frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{(x^3 + 6)^2}.
\end{aligned}$$

Problem 9. (10 pts) Find the derivative of $f(x) = \sec x \tan x$.

Solution.

$$\begin{aligned}
f'(x) &= (\sec x \tan x)' \\
&= (\sec x)'(\tan x) + (\sec x)(\tan x)' \\
&= (\sec x \tan x)(\tan x) + (\sec x)(\sec^2 x) \\
&= \sec x \tan^2 x + \sec^3 x \\
&= \sec x(\tan^2 x + \sec^2 x).
\end{aligned}$$

Problem 10. (10 pts) Find the derivative of $f(x) = \sin^{-1}(2x)$.

Solution.

$$\begin{aligned} f'(x) &= (\sin^{-1}(2x))' \\ &= \frac{1}{\sqrt{1-(2x)^2}}(2x)' \\ &= \frac{1}{\sqrt{1-4x^2}}(2) \\ &= \frac{2}{\sqrt{1-4x^2}}. \end{aligned}$$

Problem 11. (10 pts) Find the derivative of $f(x) = \ln(x^2 + 1)$.

Solution.

$$\begin{aligned} f'(x) &= (\ln(x^2+1))' \\ &= \frac{1}{(x^2+1)}(x^2+1)' \\ &= \frac{1}{(x^2+1)}(2x) \\ &= \frac{2x}{x^2+1}. \end{aligned}$$

Problem 12. (10 pts) Use implicit differentiation to compute dy/dx for the implicit function

$$x^3 + y^3 = 1,$$

showing all your work.

Solution.

$$\begin{aligned} x^3 + y^3 &= 1 \\ 3x^2 + 3y^2 \frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= \frac{-3x^2}{3y^2} = \frac{-x^2}{y^2}. \end{aligned}$$

Problem 13. (10 pts) Compute dy/dx for the function $y = \int_0^x \sqrt{1+t^2}\, dt$. Justify your answer.

Solution.

$$\frac{dy}{dx} = \sqrt{1+x^2}$$

by the Fundamental Theorem of Calculus (Part 1).

Problem 14. (20 pts) Suppose x, y, z are differentiable functions with respect to t. If $x^2 + y^2 + z^2 = 9, dx/dt = 5$, and $dy/dt = 4$, find dz/dt when $(x, y, z) = (2, 2, 1)$.

Solution. First, we have

$$\begin{aligned} x^2 + y^2 + z^2 &= 9 \\ 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt} &= 0 \\ x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt} &= 0 \\ (2)(5) + (2)(4) + (1)\frac{dz}{dt} &= 0 \\ 10 + 8 + \frac{dz}{dt} &= 0 \\ \frac{dz}{dt} &= -18. \end{aligned}$$

Problem 15. (20 pts) Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

Solution. The square of the distance from (x, y) to $(1, 4)$ is

$$(x-1)^2 + (y-4)^2.$$

This has too many variables in it, so we need to eliminate x from this function:

$$\begin{aligned}(x-1)^2 + (y-4)^2 &= \left(\frac{1}{2}y^2 - 1\right)^2 + (y-4)^2 \\ &= \frac{1}{4}y^4 - y^2 + 1 + y^2 - 8y + 16 \\ &= \frac{1}{4}y^4 - 8y + 17.\end{aligned}$$

So, we need to minimize the function

$$f(y) = \frac{1}{4}y^4 - 8y + 17.$$

Taking the derivative, we get

$$f'(y) = y^3 - 8.$$

We find the critical points.

$$\begin{aligned}y^3 - 8 &= 0 \\ y^3 &= 8 \\ y &= 2.\end{aligned}$$

This is a global minimum. So, the point on the parabola closest to $(1, 4)$ is $(2, 2)$.

Problem 16. Let
$$f(x) = x^3 - 3x + 3.$$
Find the intervals on which the function is increasing, decreasing, concave up, and concave down, as well as all local max's, min's, and inflection points. (28 pts)

You can put your work here and on the next page.

Solution. Taking the first derivative, we get
$$f'(x) = 3x^2 - 3 = 3(x-1)(x+1).$$
Using either a number line and test points or by knowing this is a parabola opening upward, $f'(x) > 0$ on the intervals $(-\infty, -1)$ and $(1, \infty)$ and $f'(x) < 0$ on the interval $(-1, 1)$. So, the graph of $f(x)$ is increasing on the intervals $(-\infty, -1)$ and $(1, \infty)$ and decreasing on the interval $(-1, 1)$. By the First Derivative Test, f has a relative maximum at $x = -1$ and a relative minimum at $x = 1$.

Taking the second derivative, we get
$$f''(x) = 6x.$$
It's easy to see that $f''(x) > 0$ if $x > 0$ and $f''(x) < 0$ if $x < 0$. So, the graph of $f(x)$ is concave up on the interval $(0, \infty)$ and concave down on the interval $(-\infty, 0)$. The graph of f has an inflection point at $x = 0$.

(a) The function is increasing on the interval(s) $(-\infty, -1)$ and $(1, \infty)$

(b) The function is decreasing on the interval(s) $(-1, 1)$

(c) The function has (a) local max(es) at the point(s) $x = -1$

(d) The function has (a) local min(s) at the point(s) $x = 1$

(e) The function is concave up on the interval(s) $(0, \infty)$

(f) The function is concave down on the interval(s) $(-\infty, 0)$

(g) The function has (an) inflection point(s) at $x = 0$

Problem 17. (10 pts) $\int \left(3x^2 + \frac{x}{2}\right)\,dx$

Solution.

$$\int \left(3x^2 + \frac{x}{2}\right)\,dx = x^3 + \frac{1}{4}x^2 + C,$$

by inspection.

Problem 18. (10 pts) $\int 3\sec^2 x\,dx$

Solution.

$$\int 3\sec^2 x\,dx = 3\tan x + C,$$

by inspection.

Problem 19. (10 pts) $\int_0^1 \frac{1}{1+x^2}\,dx$

Solution.

$$\begin{aligned}\int \frac{1}{1+x^2}\,dx &= \arctan x\big|_0^1\\ &= \arctan 1 - \arctan 0\\ &= \frac{\pi}{4}.\end{aligned}$$

Problem 20. (10 pts) $\int_0^{\pi/2} \sin^3 x \cos x\,dx$

Solution. Let $u = \sin x$. Then $du = \cos x\,dx$. Substituting, we get

$$\begin{aligned}\int_0^{\pi/2} \sin^3 x \cos x\,dx &= \int_0^1 u^3\,du\\ &= \frac{1}{4}u^4\Big|_0^1\\ &= \frac{1}{4}.\end{aligned}$$