# Integration Formulas and the Net Change Theorem

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# Outline

# Basic Integration Formulas

Recall the integration formulas given in the Table in Antiderivatives (p. 489 in the text book) and the rule on properties of definite integrals. Let's look at a few examples of how to apply these rules.

# Basic Integration Formulas

## Example

Use basic integration formulas to compute

$$\int \left( \sqrt{x} - \frac{1}{\sqrt{x}} \right) \, dx.$$

# Basic Integration Formulas

## Solution

*We use the power rule and sum rule for integrals.*

$$\begin{aligned}
\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)\, dx &= \int \left(x^{1/2} - x^{-1/2}\right)\, dx \\
&= \int x^{1/2}\, dx - \int x^{-1/2}\, dx \\
&= \frac{x^{3/2}}{3/2} - \frac{x^{1/2}}{1/2} + C \\
&= \frac{2}{3}x^{3/2} - 2x^{1/2} + C.
\end{aligned}$$

# The Net Change Theorem

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**Theorem 5.6: Net Change Theorem**

The new value of a changing quantity equals the initial value plus the integral of the rate of change:

$$F(b) = F(a) + \int_a^b F'(x)\,dx$$

or

$$\int_a^b F'(x)\,dx = F(b) - F(a).$$

# Example

## Example

Find the net displacement and total distance traveled in meters given the velocity function $f(t) = \frac{1}{2}e^t - 2$ over the interval $[0, 2]$.

# Example

## Solution

*The net displacement is given by the integral of the velocity function from $t = 0$ to $t = 2$.*

$$\begin{aligned}\int_0^2 \frac{1}{2}e^t - 2\,dt &= \frac{1}{2}e^t - 2t\Big|_0^2 \\ &= \left(\frac{1}{2}e^2 - 2(2)\right) - \left(\frac{1}{2}e^0 - 2(0)\right) \\ &= \frac{1}{2}e^2 - \frac{9}{2}.\end{aligned}$$

# Integrating Even and Odd Functions

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## Rule: Integrals of Even and Odd Functions

For continuous even functions such that $f(-x) = f(x)$,

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx.$$

For continuous odd functions such that $f(-x) = -f(x)$,

$$\int_{-a}^{a} f(x)\,dx = 0.$$

# Example

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Integrate the function

$$\int_{-2}^{2} x^4 \, dx.$$

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Since $f(x) = x^4$ is an even function, we simply apply the rule for even functions:

$$\int_{-2}^{2} x^4 \, dx = 2 \int_{0}^{2} x^4 \, dx = 2 \left( \frac{1}{5}(2)^5 - \frac{1}{5}(0)^5 \right) = \frac{64}{5}.$$