

Max-Min Worksheet 1

Problem. A farmer has 200 yd of fence with which to construct three sides of a rectangular pen; an existing long, straight wall will form the fourth side. What dimensions will maximize the area of the pen?

Solution. First, draw a picture:

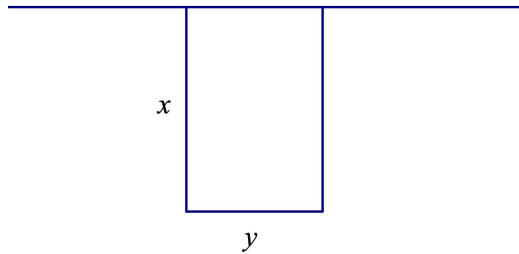


Figure 1: $y = f(t)$

We wish to maximize the area of the pen. Using the variables in Figure 1, we see that the area of the rectangular pen is

$$A = xy. \tag{1}$$

This has too many variables in it, so we need a relationship between x and y . We are told that the farmer has 200 yd of fence to fence three sides of the pen. So,

$$2x + y = 200.$$

Solving this equation for y

$$\begin{aligned} 2x + y &= 200 \\ y &= 200 - 2x \end{aligned}$$

and substituting into Equation (1), we get

$$\begin{aligned} A &= xy \\ &= x(200 - 2x). \end{aligned}$$

Since both x and y must be greater than (or equal to) zero, we have $0 \leq x \leq 100$. We must maximize this function on this interval.

Taking the derivative, we get

$$\frac{dA}{dx} = 200 - 4x$$

and setting this equal to zero to find critical points, we find that the only critical point is $x = 50$.

We know that this continuous function on this closed interval must have both a maximum value and a minimum value. By Fermat's Theorem, these values can only occur at the critical point(s) or the endpoints. Since A is zero at both endpoints and $A(50) = 5000$, this is the maximum value of A on the interval $[0, 100]$.

The problem asks for the dimensions of the pen. One dimension is $x = 50$ yd. The other dimension is $y = 200 - 2x = 200 - 2(50) = 100$ yd. So, the dimensions of the largest pen is 50 yd \times 100 yd.