Test #2
MATH 1634

March 13, 2026

DIRECTIONS: *This is the second test for this section of MATH 1634. The test contains ten problems counting ten points each for a total of one hundred points. You must complete all the problems.* **You may NOT use your calculator.** *You must show all your work clearly and completely in the spaces provided. You may not use your book or your notes. You may not give help to or receive help from anyone else. To do so will result in you receiving a grade of F in the course.*

Good luck.

My signature below indicates that I have read and understand the instructions printed above and I agree to abide by them.

Name (printed):_____

DIRECTIONS: *Compute dy/dx for each of the following functions. Good luck.*

Problem 1. $y = x \sin x$

Solution.

$$\frac{dy}{dx} = \sin x + x \cos x.$$

Problem 2. $y = \dfrac{\ln x}{x}$

Solution.

$$\begin{aligned}\frac{dy}{dx} &= \frac{[\ln x]' x - \ln x [x]'}{x^2} \\ &= \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} \\ &= \frac{1 - \ln x}{x^2}.\end{aligned}$$

Problem 3. $y = (3x^2 + 4x - 3)^8$

Solution.

$$\begin{aligned}\frac{dy}{dx} &= 8(3x^2 + 4x - 3)^7(3x^2 + 4x - 3)' \\ &= 8(3x^2 + 4x - 3)^7(6x + 4) \\ &= 8(6x + 4)(3x^2 + 4x - 3)^7 \\ &= 16(3x + 2)(3x^2 + 4x - 3)^7.\end{aligned}$$

Problem 4. $y = \ln(\tan x)$

Solution.

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{\tan x}(\tan x)' \\ &= \frac{1}{\tan x}(\sec^2 x) \\ &= \frac{\sec^2 x}{\tan x} \\ &= \sec x \csc x.\end{aligned}$$

Problem 5. $y = \log_5(x^2)$

Solution 1.

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{x^2 \ln 5}[x^2]' \\ &= \frac{1}{x^2 \ln 5} 2x \\ &= \frac{2x}{x^2 \ln 5} \\ &= \frac{2}{x \ln 5}.\end{aligned}$$

Solution 2.

$$\begin{aligned}y &= \log_5(x^2) \\ &= 2\log_5(x) \\ \frac{dy}{dx} &= 2 \cdot \frac{1}{x \ln 5} \\ &= \frac{2}{x \ln 5}.\end{aligned}$$

Problem 6. $y = 3x \sec(2x)$

Solution.

$$\begin{aligned}\frac{dy}{dx} &= [3x]' \sec(2x) + 3x[\sec(2x)]' \\ &= 3\sec(2x) + 3x \sec(2x)\tan(2x)[2x]' \\ &= 3\sec(2x) + 3x \sec(2x)\tan(2x) \cdot 2 \\ &= 3\sec(2x) + 6x \sec(2x)\tan(2x).\end{aligned}$$

Problem 7. $y = xe^{x^2}$

Solution.

$$\begin{aligned}\frac{dy}{dx} &= [x]'e^{x^2} + x[e^{x^2}]' \\ &= (1)e^{x^2} + xe^{x^2}[x^2]' \\ &= e^{x^2} + xe^{x^2}(2x) \\ &= e^{x^2} + 2x^2e^{x^2} \\ &= e^{x^2}(1 + 2x^2).\end{aligned}$$

Problem 8. $x^2 + xy + y^2 = 48$

Solution.

$$\begin{aligned}x^2 + xy + y^2 &= 48 \\ 2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} &= 0 \\ (x + 2y)\frac{dy}{dx} &= -2x - y \\ \frac{dy}{dx} &= \frac{-2x - y}{x + 2y}.\end{aligned}$$

Problem 9. $y = x\cos^{-1} x - \sqrt{1-x^2}$. Simplify your answer.

Solution.

$$\begin{aligned}
\frac{dy}{dx} &= (x)'\cos^{-1} x + x(\cos^{-1} x)' - [(1-x^2)^{1/2}] \\
&= (1)\cos^{-1} x + x \cdot \frac{-1}{\sqrt{1-x^2}} - \left[\frac{1}{2}(1-x^2)^{-1/2}\right](-2x) \\
&= \cos^{-1} x - \frac{x}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}} \\
&= \cos^{-1} x.
\end{aligned}$$

Problem 10. Use logarithmic differentiation to compute dy/dx.

$$y = \sqrt{(x^2+1)(x-1)^2}, \quad x > 1.$$

Solution.

$$\begin{aligned}
y &= \sqrt{(x^2+1)(x-1)^2} = [(x^2+1)(x-1)^2]^{1/2} = (x^2+1)^{1/2}(x-1) \\
\ln y &= \ln[(x^2+1)^{1/2}(x-1)] \\
&= \ln[(x^2+1)^{1/2}] + \ln(x-1) \\
&= \frac{1}{2}\ln(x^2+1) + \ln(x-1) \\
\frac{1}{y}\frac{dy}{dx} &= \frac{1}{2}\cdot\frac{1}{x^2+1}\cdot 2x + \frac{1}{x-1}(1) \\
&= \frac{x}{x^2+1} + \frac{1}{x-1} \\
\frac{dy}{dx} &= y\left[\frac{x}{x^2+1} + \frac{1}{x-1}\right] \\
&= \sqrt{(x^2+1)(x-1)^2}\left[\frac{x}{x^2+1} + \frac{1}{x-1}\right] \\
&= \frac{x(x-1)}{\sqrt{x^2+1}} + \sqrt{x^2+1}.
\end{aligned}$$

The penultimate line is an acceptable answer.