Max-Min Worksheet 5

Problem. A farmer wants to hire workers to pick 900 bushels of beans. Each worker can pick 5 bushels per hour and is paid \$1.00 per bushel. The farmer must also pay a supervisor \$10 per hour while the picking is in progress, and he has additional miscellaneous expenses of \$8 per worker. How many workers should he hire to minimize the total cost? What will then be the cost per bushel picked?

Solution. Let x be the number of workers and y be the number of hours worked.

Each worker can pick 5-bushels per hour and is paid \$1.00 per bushel. So, each worker is paid \$5.00 per hour. So, the cost of x workers per hour is $5x$ dollars and the cost of all y hours is $5xy$. The supervisor costs \$10 per hour, so the cost of the supervisor is $10y$ dollars. The miscellaneous expenses are \$8.00 per worker, so the miscellaneous expenses for x workers is $8x$ dollars. So, the total cost is $C = 5xy + 10y + 8x$.

This is the function we want to minimize. This function has too many variables, so we need a relationship between the variables.

We are told there are 900 bushels of beans to pick. Each worker can pick 5-bushels per hour. So, each worker can pick $5y$ bushels in y hours. So, all x workers can pick $5xy$ bushels in y. Thus, we have $5xy = 900$, or $xy = 180$.

Solving this for y

$$y = \frac{180}{x}$$

and substituting into the cost function, we get

$$\begin{aligned} C &= 5xy + 10y + 8x \\ &= 5x\left(\frac{180}{x}\right) + 10\left(\frac{180}{x}\right) + 8x \\ &= 900 + \frac{1800}{x} + 8x. \end{aligned}$$

For the interval in which we are interested, all we know is that $x \geq 0$. So, we don't have a continuous function on a closed interval, so we'll have to be a bit careful.

Taking the derivative, we get

$$C' = -\frac{1800}{x^2} + 8,$$

and setting this equal to zero and solving gives us our only critical point in the interval $(0,\infty)$: $x = 15$.

Choosing test points in the intervals $(0,15)$ and $(15,\infty)$, we see that $C' < 0$ on the interval $(0,15)$ and $C' > 0$ on the interval $(15,\infty)$. Hence $x = 15$ is a global minimum for the cost function on the interval $(0,\infty)$.

So, in order to minimize the total cost of picking the beans, there should be 15 workers hired and the total cost of the picking will then be

$$C(15) = 900 + \frac{1800}{15} + 8(15) = \$1140.$$

So, the cost per bushel picked is $1140/900 \approx \$1.27$.