Max-Min Worksheet 3

Problem. Suppose that the cost of publishing a small book is \$10,000 to set up the (annual) press run plus \$8 for each book printed. The publisher sold 7000 copies last year at \$13 each, but sales dropped to 5000 copies this year when the price was raised to \$15 per copy. Assume that up to 10,000 copies can be printed in a single press run. How many copies should be printed, and what should be the selling price of each copy, to maximize the year's profit on this book?

Solution. We wish to maximize profit P. Profit P is revenue R minus cost C and R is the number of books sold x times the price charged $p(x)$. The function $p(x)$ is called the **demand function**.

First, we compute the demand function. We are told that if $x = 7000$ copies are sold, the price is $p(x) = 13$ dollars. We are also told that is $x = 5000$ copies are sold, the price is $p(x) = 15$ dollars. Assuming this relationship is linear, we need the equation of a line passing through $(5000, 15)$ and $(7000, 13)$. The equation of this line is $y - 15 = -\frac{1}{1000}(x - 5000)$, or

$$p(x) = -\frac{1}{1000}x + 20.$$

From this, we compute the revenue function

$$R(x) = xp(x) = x\left(-\frac{1}{1000}x + 20\right)$$

Next, we need the cost function. The cost of publishing the books is the fixed cost plus the variable cost. The fixed cost is given as 10000. The variable cost is given as 8 dollars per book, so the variable cost of publishing x books is $8x$ dollars. So, the total cost is

$$C(x) = 10000 + 8x.$$

Now, we can compute the profit function

$$\begin{aligned} P(x) = R(x) - C(x) &= x\left(-\frac{1}{1000}x + 20\right) - (10000 + 8x) \\ &= -\frac{1}{1000}x^2 + 12x - 10000. \end{aligned}$$

We are told that up to 10000 copies can be made in one run, so $0 \leq x \leq 10000$. We must maximize this function on this interval.

Taking the derivative

$$P'(x) = -\frac{1}{500}x + 12$$

and setting it equal to zero, we find that $P'(x) = 0$ if $x = 6000$. Notice that P is a quadratic polynomial with a negative leading coefficient, so its vertex—at $x = 6000$—is a global maximum. In order to sell 6000 books, the price must be $p(6000) = \$14$.

So, to maximize the profit, the publisher should publish 6000 copies of the book and sell them for a price of fourteen dollars each.