

Trigonometry Formulas

1. Fundamental Trigonometric Identities

a. Pythagorean Identities

$$\bullet \sin^2 \theta + \cos^2 \theta = 1 \qquad \bullet 1 + \tan^2 \theta = \sec^2 \theta \qquad \bullet 1 + \cot^2 \theta = \csc^2 \theta$$

b. Even-Odd Identities

$$\begin{aligned} \bullet \sin(-x) &= -\sin(x) & \bullet \cos(-x) &= \cos(x) & \bullet \tan(-x) &= -\tan(x) \\ \bullet \csc(-x) &= -\csc(x) & \bullet \sec(-x) &= \sec(x) & \bullet \cot(-x) &= -\cot(x) \end{aligned}$$

c. Cofunction Identities

$$\begin{aligned} \bullet \sin\left(\frac{\pi}{2} - \theta\right) &= \cos(\theta) & \bullet \cos\left(\frac{\pi}{2} - \theta\right) &= \sin(\theta) \\ \bullet \tan\left(\frac{\pi}{2} - \theta\right) &= \cot(\theta) & \bullet \cot\left(\frac{\pi}{2} - \theta\right) &= \tan(\theta) \\ \bullet \sec\left(\frac{\pi}{2} - \theta\right) &= \csc(\theta) & \bullet \csc\left(\frac{\pi}{2} - \theta\right) &= \sec(\theta) \end{aligned}$$

d. Reciprocal Identities

$$\begin{aligned} \bullet \sin(\theta) &= \frac{1}{\csc(\theta)} & \bullet \cos(\theta) &= \frac{1}{\sec(\theta)} & \bullet \tan(\theta) &= \frac{1}{\cot(\theta)} \\ \bullet \csc(\theta) &= \frac{1}{\sin(\theta)} & \bullet \sec(\theta) &= \frac{1}{\cos(\theta)} & \bullet \cot(\theta) &= \frac{1}{\tan(\theta)} \end{aligned}$$

e. Quotient Identities

$$\bullet \tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \bullet \cot \theta = \frac{\cos \theta}{\sin \theta}$$

f. Periodic Properties of Sine, Cosine, Secant, and Cosecant Functions

- $\sin(\theta + 2\pi) = \sin(\theta)$
- $\csc(\theta + 2\pi) = \csc(\theta)$
- $\cos(\theta + 2\pi) = \cos(\theta)$
- $\sec(\theta + 2\pi) = \sec(\theta)$

g. Periodic Properties of Tangent and Cotangent

- $\tan(\theta + \pi) = \tan(\theta)$
- $\cot(\theta + \pi) = \cot(\theta)$

2. Sum and Difference Formulas

- $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
- $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
- $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$
- $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$
- $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$
- $\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

3. Double-Angle Formulas

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 1 - 2 \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta}\end{aligned}$$

4. Half-Angle Formula for Tangent

$$\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}$$

5. Power-Reducing Formulas

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

6. Product-to-Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

7. Sum-to-Product Formulas

$$\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

$$\sin \alpha - \sin \beta = 2 \sin \left(\frac{\alpha - \beta}{2} \right) \cos \left(\frac{\alpha + \beta}{2} \right)$$

$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

$$\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)$$