4.2 Modeling With Linear Functions

Chapter 4 Linear Functions

Concepts & Objectives

- Objectives for this section are
  - Build linear models from verbal descriptions.
  - Model a set of data with a linear function.

Verbal Description to Linear Model

- When building linear models to solve problems involving quantities with a constant rate of change, we typically follow the same problem strategies that we would use for any type of function.
- In these problems, we will either be given the rate of change (i.e. the slope) or at least two points to use to calculate the slope.
- Plug this information into the point-slope form of a line and simplify to arrive at your function.

$$y - y_1 = m(x - x_1) \Rightarrow f(x) = m(x - x_1) + y_1$$

Verbal Description to Linear Model

- Example: A town's population has been growing at a constant rate. In 2004, the population was 6,200. By 2009, the population had grown to 8,100. Assume this trend continues.
  - Predict the population in 2013.

Verbal Description to Linear Model

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  - Predict the population in 2013.

    First, we have to create a function. We have two points of data: (2004, 6200) and (2009, 8100). From this we can calculate the slope:

$$m = \frac{8100 - 6200}{2009 - 2004} = \frac{1900}{5} = 380$$

Verbal Description to Linear Model

- Example: A town's population has been growing at a constant rate. In 2004, the population was 6,200. By 2009, the population had grown to 8,100. Assume this trend continues.
  - Predict the population in 2013.

    Now we can plug in one of our data points to create the function:

$$P(t) = 380(t - 2009) + 8100$$
$$= 380t - 755320$$

Verbal Description to Linear Model

- Example: A town's population has been growing at a constant rate. In 2004, the population was 6,200. By 2009, the population had grown to 8,100. Assume this trend continues.
  - Predict the population in 2013.

    Plug in 2013 for t to get the population:

$$P(2013) = 380(2013) - 755320$$

$$= 9{,}620 \text{ in } 2013$$

Verbal Description to Linear Model

- Example: A town's population has been growing at a constant rate. In 2004, the population was 6,200. By 2009, the population had grown to 8,100. Assume this trend continues.
  - Predict the population in 2013.

    Because these numbers can get pretty big, it can be easier to define t as the number of years since 2004. Which makes 2009: $t = 5$ and 2013: $t = 9$

$$m = \frac{8100 - 6200}{5 - 0} = 380 \qquad P(t) = 380t + 6200$$

Verbal Description to Linear Model

- Example: A town's population has been growing at a constant rate. In 2004, the population was 6,200. By 2009, the population had grown to 8,100. Assume this trend continues.
  - Predict the population in 2013.

    This makes our problem:

$$P(9) = 380(9) + 6200$$

$$= 9{,}620 \text{ in } 2013$$

Verbal Description to Linear Model

- Example: A town's population has been growing at a constant rate. In 2004, the population was 6,200. By 2009, the population had grown to 8,100. Assume this trend continues.
  - When will the population reach 15,000?

    Using the previous definition of *t*, we can set the function equal to 15,000 and solve *t*.

$$380t + 6200 = 15000$$

$$380t = 8800$$

$$t \approx 23.16 \text{ years after 2004}$$

$$\text{or } 2027$$

Geometry and Linear Functions

- Since linear functions create graphs of lines, intersections of collections of linear functions can create geometric shapes.
- If you are asked to find the area created by some linear functions:
  - Graph the functions (using Desmos is easiest)
  - Triangle and parallelogram area formulas require that you find the height and the base of the shape.
    - These are not always going to be vertical and horizontal, but they must be perpendicular to each other.

Geometry and Linear Functions

- Example: Find the area of the triangle bounded by the x-axis, the line $f(x) = -\frac{3}{4}x + 15$, and the line perpendicular to $f(x)$ that goes through the origin.

Geometry and Linear Functions

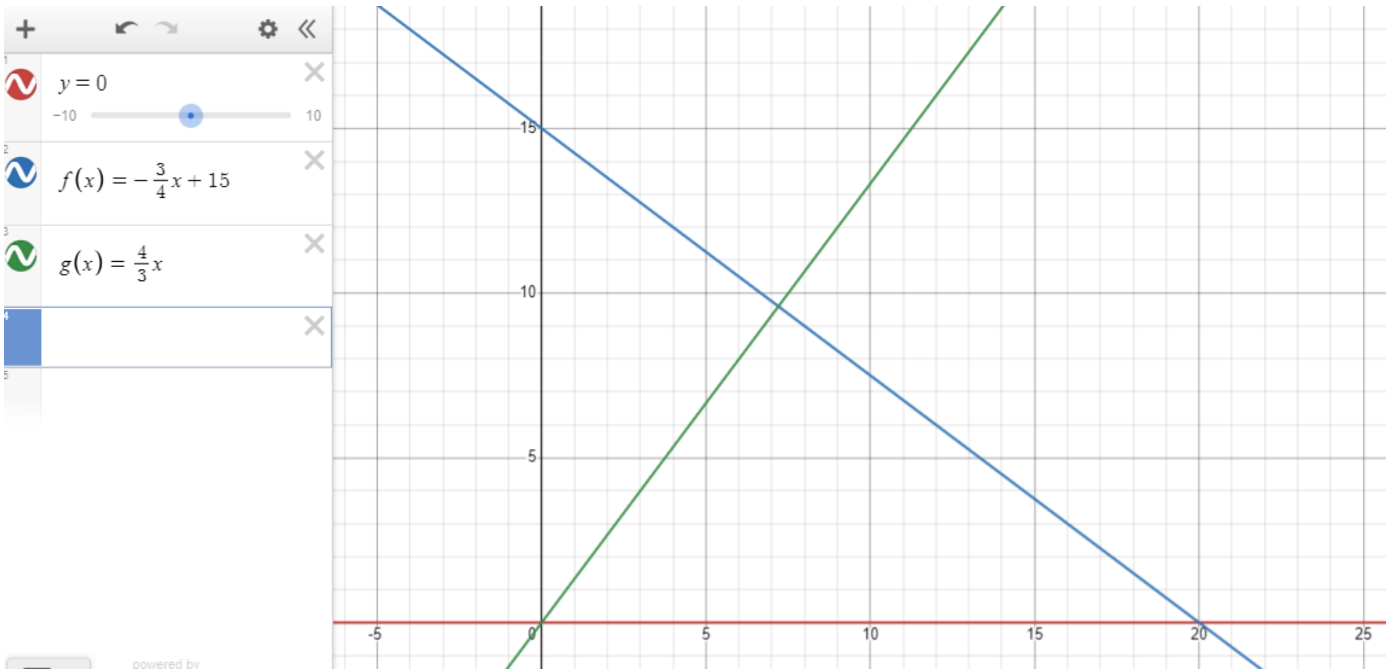
- Example: Find the area of the triangle bounded by the x-axis, the line $f(x) = -\frac{3}{4}x + 15$, and the line perpendicular to $f(x)$ that goes through the origin.

  First, we have to find the third line, which we'll call $g(x)$. Recall that the perpendicular slope is the negative reciprocal of the other slope:

$$g(x) = \frac{4}{3}(x - 0) + 0 \Rightarrow g(x) = \frac{4}{3}x$$

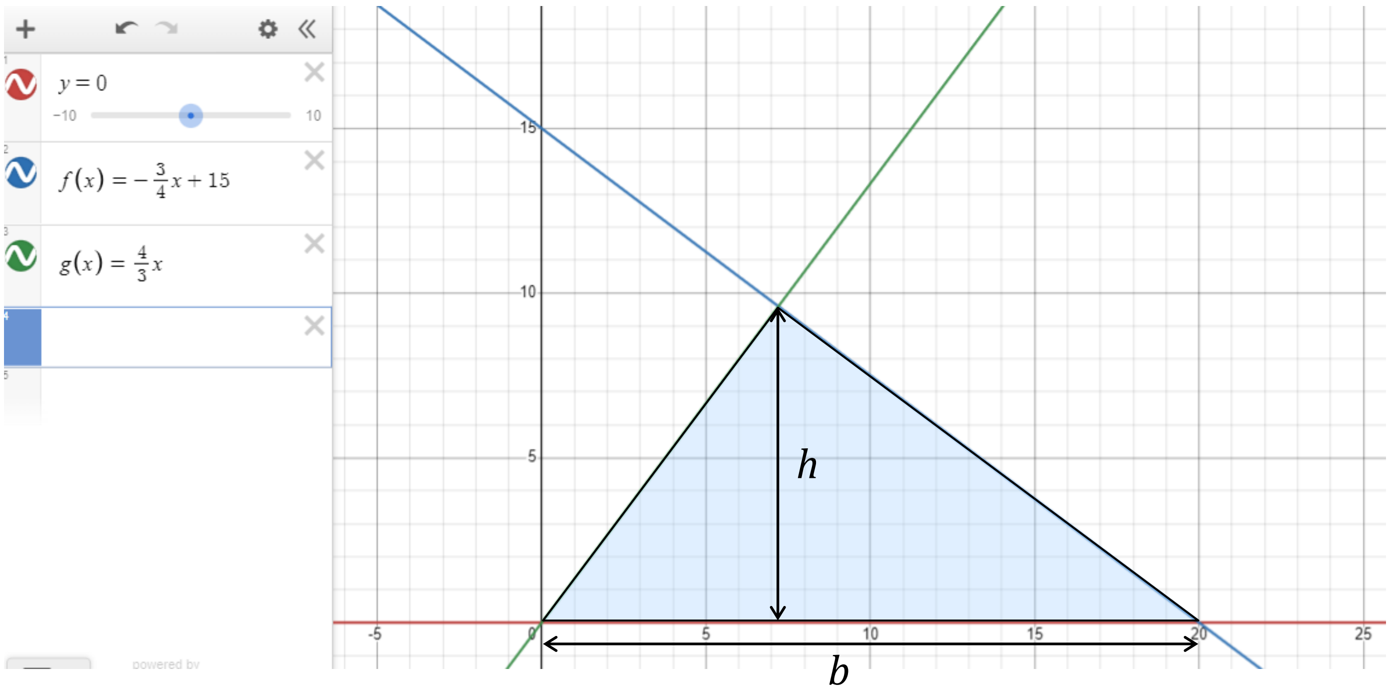
Geometry and Linear Functions

- Now we can graph these in Desmos, and see what this shape looks like (the *x*-axis is the line *y* = 0):



Geometry and Linear Functions

- This is the triangle whose area we are trying to find:



Geometry and Linear Functions

- We can see that the base is 20 units, but the height is a little trickier. We need the *y*-coordinate of the intersection.
- There are two ways to find the intersection point
  - Set the two functions equal to each other and solve for *x* and find *y*.

Geometry and Linear Functions

- There are two ways to find the intersection point
  - Set the two functions equal to each other and solve for *x* and find *y*.

$$\frac{4}{3}x = -\frac{3}{4}x + 15$$

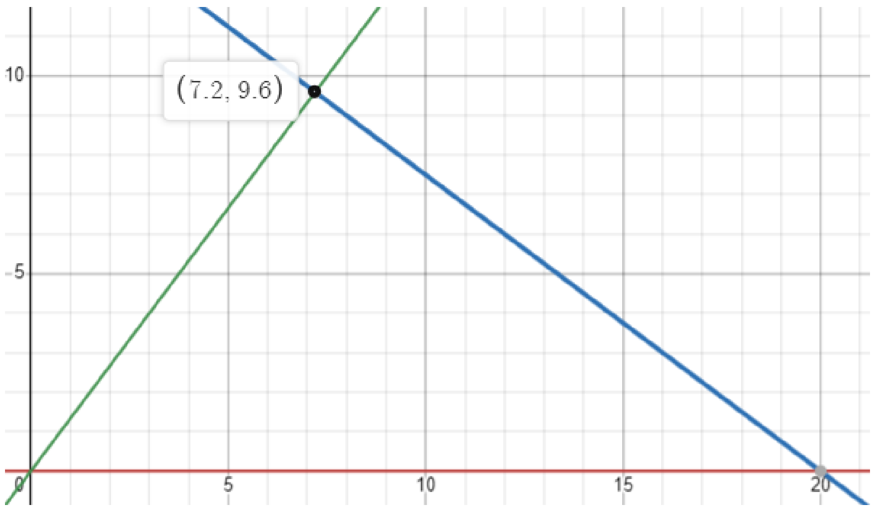
$$\frac{3}{4}x + \frac{4}{3}x = 15$$

$$\frac{25}{12}x = 15$$

$$x = 15\left(\frac{12}{25}\right) = \frac{36}{5} = 7.2 \qquad y = \frac{4}{3}(7.2) = 9.6$$

Geometry and Linear Functions

- There are two ways to find the intersection point
  - Click on the intersection in Desmos:

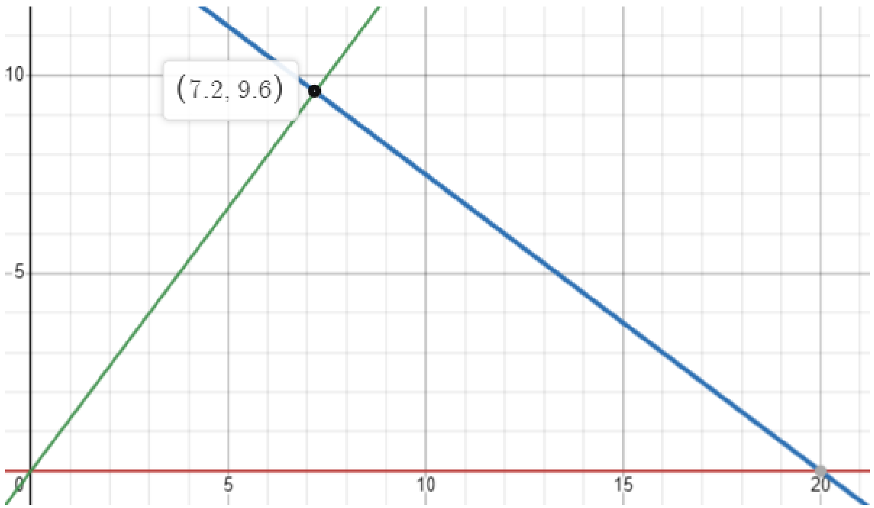


(Obviously, the Desmos method is easier, but sometimes you might need a fraction as an answer, which Desmos does not provide.)

  - So, the height of the triangle is 9.6 units.

Geometry and Linear Functions

- Now that we have the base and height, we can find the area of the triangle.



$$A = \frac{bh}{2}$$

$$= \frac{(20)(9.6)}{2}$$

$$= 96 \text{ sq. units}$$

Modeling With Linear Functions

- Real-world situations including two or more linear functions may be modeled with a system of linear equations.
- Find the solution by setting two linear functions equal to each other and solving for x, or find the point of intersection on a graph.

Modeling With Linear Functions

- Example: Jamal is choosing between two truck-rental companies. The first, Keep on Trucking, Inc., charges an up-front fee of $20, then 59 cents per mile. The second, Move It Your Way, charges an up-front fee of $16, then 63 cents per mile. When will Keep on Trucking, Inc. be the better choice for Jamal?

Modeling With Linear Functions

- Example: Jamal is choosing between two truck-rental companies. The first, Keep on Trucking, Inc., charges an up-front fee of $20, then 59 cents per mile. The second, Move It Your Way, charges an up-front fee of $16, then 63 cents per mile. When will Keep on Trucking, Inc. be the better choice for Jamal?

    In each case, there is a **constant** term ($20 or $16), and a **rate** (59¢ or 63¢). The rate goes with the variable; that is our slope.

Modeling With Linear Functions

- Example: Jamal is choosing between two truck-rental companies. The first, Keep on Trucking, Inc., charges an up-front fee of $20, then 59 cents per mile. The second, Move It Your Way, charges an up-front fee of $16, then 63 cents per mile. When will Keep on Trucking, Inc. be the better choice for Jamal?

Input	d, distance driven in miles
Outputs	$K(d)$: cost, in dollars, for renting from Keep on Trucking $M(d)$ cost, in dollars, for renting from Move It Your Way
Initial Value	Up-front fee: $K(0) = 20$ and $M(0) = 16$
Rate of Change	$K(d) = \$0.59$ /mile and $P(d) = \$0.63$ /mile

Modeling With Linear Functions

- Example: Jamal is choosing between two truck-rental companies. The first, Keep on Trucking, Inc., charges an up-front fee of $20, then 59 cents per mile. The second, Move It Your Way, charges an up-front fee of $16, then 63 cents per mile. When will Keep on Trucking, Inc. be the better choice for Jamal?

Driving 0 miles gives us the *y*-intercept:

$$K(d) = 0.59d + 20$$

$$M(d) = 0.63d + 16$$

Modeling With Linear Functions

- Example: Jamal is choosing between two truck-rental companies. The first, Keep on Trucking, Inc., charges an up-front fee of $20, then 59 cents per mile. The second, Move It Your Way, charges an up-front fee of $16, then 63 cents per mile. When will Keep on Trucking, Inc. be the better choice for Jamal?

Keep on Trucking will be the better choice when

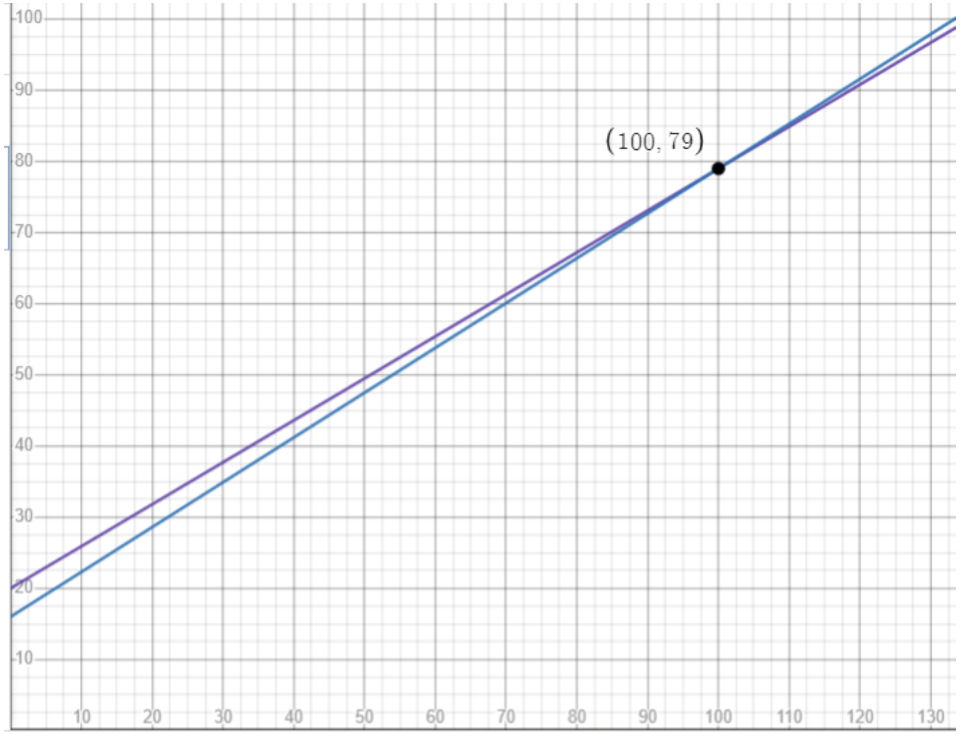
$K(d) < M(d)$ or $0.59d + 20 < 0.63d + 16$

$$-0.04d < -4$$

$$d > 100 \text{ miles}$$

Modeling With Linear Functions

- Once you have your functions set up, you can also graph the functions and find the intersection point:



Classwork

- *College Algebra 2e*
  - 4.2: 6,8,12-15 (all); 4.1: 40-68 (×4); 3.7: 38-46 (even)
  - 4.2 Classwork Check
- Quiz 4.1