# Newton's Method

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# Outline

# Newton's Method

- As usual, you should read section 4.7 in the online textbook.
- This slideshow will give an overview and an explanation of the important concepts in the book.
- This slideshow will also include a limited number of examples.
- The main purpose of this slideshow is to give an extended explanation and clarification of the material in the text.

# Newton's Method

Newton's Method is a numerical technique for finding roots of equations. This technique is a part of the mathematical discipline called numerical analysis.

The idea is to start at a point near a root and construct the tangent line at that point on the curve. You then find the root of the tangent line. You then repeat the process iteratively. When the numbers produced approach a limit, that's the root of the equation.

# Procedure for Newton's Method

Start with a function $f(x)$ for which we want to find a root.

Start with a point $x_0$ near a root. The point on the curve then is $(x_0, f(x_0))$ and the equation of the tangent line is

$$y = f(x_0) + f'(x_0)(x - x_0)$$

We set $y$ equal to zero to find the root of this equation.

$$x = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

Call this point $x_1$.

# Procedure for Newton's Method

Repeating this iteratively, we get the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

# Newton's Method

1. Guess a first approximation to a solution of the equation $f(x) = 0$. A graph of $y = f(x)$ may help.
2. Use the first approximation to get a second, the second to get a third, and so on, using the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

# Example

## Example

Approximate the positive root of the equation $f(x) = x^2 - 5 = 0$.

## Solution

*We start with a guess near the root of* $f$. *Let's take* $x_0 = 2$. *Now we use the formula*

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 5}{2x_n}.$$

*to compute* $x_1$, $x_2$, $x_3$, *and so on. We compute:*

| | |
|---|---|
| $x_0$ | 2 |
| $x_1$ | 2.25 |
| $x_2$ | 2.23611 |
| $x_3$ | 2.236067978 |
| $x_4$ | 2.236067978 |

*Once the numbers stop changing, this is the root.*

# Comments on Newton's Method

I have a few comments on Newton's Method.

First, it doesn't work all the time. The expression $f'(x_n)$ can't be zero at any point. Further, it's possible for the iteration to fail to produce a limit. He can get caught in a loop and repeat over and over.

Second, when Newton's method does work—which is most of the time—it works very quickly. Once there is one decimal place of accuracy, the number of correct decimal places doubles with every iteration. So, after you get one correct decimal place, the next iteration will produce two correct decimal places, the next will produce four correct decimal places, and the next will produce eight correct decimal places—which is probably all the digits your calculator shows.