Problem Set #9
Due Thursday, October 16

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Problem 4.1.1. Label the following statements as true or false.

(a) The function $\det : M_{2\times 2}(\mathbb{F}) \to \mathbb{F}$ is a linear transformation.

(b) The determinant of a 2×2 matrix is a linear function of each row of the matrix when the other row is held fixed.

(c) If $A \in M_{2\times 2}(\mathbb{F})$ and $\det(A) = 0$, then A is invertible.

(d) If u and v are vectors in $\mathbb{R}^2$ emanating from the origin, then the area of the parallelogram having u and v as adjacent sides is

$$\det\begin{pmatrix} u \\ v \end{pmatrix}$$

(e) A coordinate system is right-handed if and only if its orientation equals 1.

Problem 4.1.5. Prove that if B is the matrix obtained by interchanging the rows of a 2×2 matrix A, then $\det(B) = -\det(A)$.

Problem 4.1.6. Prove that if the two columns of $A \in M_{2\times 2}(\mathbb{F})$ are identical, then $\det(A) = 0$.

Problem 4.1.7. Prove that $\det\left(A^t\right) = \det(A)$ for any $A \in M_{2\times 2}(\mathbb{F})$.

Problem 4.1.10. The **classical adjoint** of a 2×2 matrix $A \in M_{2\times 2}(\mathbb{F})$ is the matrix

$$C = \begin{pmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{pmatrix}$$

Prove that

(a) $CA = AC = [\det(A)]I$.

(b) $\det(C) = \det(A)$.

(c) The classical adjoint of A^t is C^t.

(d) If A is invertible, then $A^{-1} = [\det(A)]^{-1}C$.