Problem Set #8
Due Thursday October 9

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Problem 3.4.1. Label the following statements as true or false.

(a) If $(A'|b')$ is obtained from $(A|b)$ by a finite sequence of elementary column operations, then the systems $Ax = b$ and $A'x = b'$ are equivalent.

(b) If $(A'|b')$ is obtained from $(A|b)$ by a finite sequence of elementary row operations, then the systems $Ax = b$ and $A'x = b'$ are equivalent.

(c) If A is an $n \times n$ matrix with rank n, then the reduced row echelon form of A is I_n.

(d) Any matrix can be put in reduced row echelon form by means of a finite sequence of elementary row operations.

(e) If $(A|b)$ is in reduced row echelon form, then the system $Ax = b$ is consistent.

(f) Let $Ax = b$ be a system of m linear equations in n unknowns for which the augmented matrix is in reduced row echelon form. If this system is consistent, the the dimension of the solution set of $Ax = 0$ is $n - r$, where r equals the number of nonzero rows in A.

(g) If a matrix A is transformed by elementary row operations into a matrix A' in reduced row echelon form, then the number of nonzero rows in A' equals the rank of A.

Problem 3.4.2. Use Gaussian elimination to solve the following systems of linear equations.

(b) $\begin{cases} x_1 - 2x_2 - x_3 = 1 \\ 2x_1 - 3x_2 + x_3 = 6 \\ 3x_1 - 5x_2 \quad\quad = 7 \\ x_1 \quad\quad + 5x_3 = 9 \end{cases}$

(f) $\begin{cases} x_1 + 2x_2 - x_3 + 3x_4 = 2 \\ 2x_1 + 4x_2 - x_3 + 6x_4 = 5 \\ \quad\quad x_2 \quad\quad + 2x_4 = 3 \end{cases}$

Problem 3.4.4. For each of the systems that follow, apply Exercise 3 to determine whether the system is consistent. If the system is consistent, find all solutions. Finally, find a basis for the solution set of the corresponding homogeneous system.

(b)

$$x_1 + x_2 - 3x_3 + x_4 = -2$$
$$x_1 + x_2 + x_3 - x_4 = 2$$
$$x_1 + x_2 - x_3 = 0$$

Problem 3.4.5. Let the reduced row echelon form of A be

$$\begin{pmatrix} 1 & 0 & 2 & 0 & -2 \\ 0 & 1 & -5 & 0 & -3 \\ 0 & 0 & 0 & 1 & 6 \end{pmatrix}$$

Determine A if the first, second, and fourth columns of A are

$$\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \text{ and } \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix},$$

respectively.