

# Problem Set #1

## Due Thursday, August 21

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**Problem 1.1.1.** Determine whether the vectors emanating from the origin and terminating at the following pairs of points are parallel.

- (a)  $(3, 1, 2)$  and  $(6, 4, 2)$
- (b)  $(-3, 1, 7)$  and  $(9, -3, -21)$
- (c)  $(5, -6, 7)$  and  $(-5, 6, -7)$
- (d)  $(2, 0, -5)$  and  $(5, 0, -2)$

**Problem 1.1.2.** Find the equations of the lines through the following pairs of points in space.

- (a)  $(3, -2, 4)$  and  $(-5, 7, 1)$

**Problem 1.1.3.** Find the equations of the plane containing the following points in space.

- (a)  $(2, -5, -1)$ ,  $(0, 4, 6)$ , and  $(-3, 7, 1)$

**Problem 1.1.6.** Show that the midpoint of the line segment joining the points  $(a, b)$  and  $(c, d)$  is  $((a + c)/2, (b + d)/2)$ .

**Problem 1.2.10.** Let  $V$  denote the set of all differentiable real-valued functions defined on the real line. Prove that  $V$  is a vector space with the operations of addition and scalar multiplication defined in Example 3.

**Problem 1.2.12.** A real-valued function  $f$  defined on the real line is called an **even function** if  $f(-t) = f(t)$  for each real number  $t$ . Prove that the set of even functions defined on the real line with the operations of addition and scalar multiplication defined in Example 3 is a vector space.

**Problem 1.2.14.** Let  $V = \{(a_1, a_2, \dots, a_n) : a_i \in \mathbb{C} \text{ for } 1 \leq i \leq n\}$ . So  $V$  is a vector space over  $\mathbb{C}$  by Example 1. Is  $V$  a vector space over the field of real numbers with the operations of coordinatewise addition and multiplication?

**Problem 1.2.22.** How many matrices are there in the vector space  $M_{m \times n}(\mathbb{Z}_2)$ ? (See Appendix C.)