Homework #8
Due Monday, October 6

Exercise 3.3.4. Assume K is compact and F is closed. Decide if the following sets are definitely compact, definitely closed, both, or neither.

(a) $K \cap F$

(b) $\overline{F^c \cup K^c}$

(c) $K \setminus F = \{x \in K : x \notin F\}$

(d) $\overline{K \cap F^c}$

Exercise 3.3.5. Decide whether the following propositions are true or false. If the claim is valid, supply a short proof, and if the claim is false, provide a counterexample.

(a) The arbitrary intersection of compact sets is compact.

(b) The arbitrary union of compact sets is compact.

(c) Let A be arbitrary, and let K be compact. Then, the intersection $A \cap K$ is compact.

(d) If $F_1 \supseteq F_2 \supseteq F_3 \supseteq F_4 \supseteq \cdots$ is a nested sequence of nonempty closed sets, then the intersection $\cap_{n=1}^{\infty} F_n \neq \emptyset$.

Exercise 4.2.5. Use Definition 4.2.1 to supply a proof for the following limit statements.

(a) $\lim_{x \to 2}(3x + 4) = 10$.

(b) $\lim_{x \to 0} x^3 = 0$.

(c) $\lim_{x \to 2}(x^2 + x - 1) = 5$.

(d) $\displaystyle\lim_{x \to 3} \frac{1}{x} = \frac{1}{3}$,

Exercise 4.2.11 (Squeeze Theorem). Let f, g, and h satisfy $f(x) \leq g(x) \leq h(x)$ for all x in some common domain A. If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} h(x) = L$ at some limit point c of A, show that $\lim_{x \to c} g(x) = L$ as well.