Homework #7
Due Monday, September 29

Exercise 2.7.5. Now that we have proved the basic facts about geometric series, supply a proof for Corollary 2.4.7:

Corollary. *The series* $\sum_{n=1}^{\infty} 1/n^p$ *converges if and only if* $p > 1$*.*

Exercise 2.7.7. (a) Show that if $a_n > 0$ and $\lim(na_n) = \ell$ with $\ell \neq 0$, then $\sum a_n$ diverges.

(b) Assume $a_n > 0$ and $\lim(n^2 a_n)$ exists. Show that $\sum a_n$ converges.

Exercise 3.2.2. Let

$$A = \left\{ (-1)^n + \frac{2}{n} : n = 1, 2, 3, \ldots \right\} \text{ and } B = \{x \in \mathbb{Q} : 0 < x < 1\}.$$

Answer the following questions for each set:

(a) What are the limit points?

(b) Is the set open? Closed?

(c) Does the set contain any isolated points?

(d) Find the closure of the set.

Exercise 3.2.4. Let A be nonempty and bounded above so that $s = \sup A$ exists.

(a) Show that $s \in \overline{A}$.

(b) Can an open set contain its supremum?

Exercise 3.2.8. Assume A is an open set and B is a closed set. Determine if the following sets are definitely open, definitely closed, both, or neither.

(a) $\overline{A \cup B}$

(b) $A \setminus B = \{x \in A : x \notin B\}$

(c) $(A^c \cup B)^c$

(d) $(A \cap B) \cup (A^c \cap B)$

(e) $(\overline{A})^c \cap \overline{A^c}$