

Homework #5

Due Monday, September 15

Exercise 2.4.1. (a) Prove that the sequence defined by $x_1 = 3$ and

$$x_{n+1} = \frac{1}{4 - x_n}$$

converges.

- (b) Now that we know $\lim x_n$ exists, explain why $\lim x_{n+1}$ must also exist and equal the same value.
- (c) Take the limit of each side of the recursive equation in part (a) of this exercise to explicitly compute $\lim x_n$.

Exercise 2.5.1. Give an example of each of the following, or argue that such a request is impossible.

- (a) A sequence that has a subsequence that is bounded but contains no subsequence that converges.
- (b) A sequence that does not contain 0 or 1 as a term but contains subsequences converging to each of these values.
- (c) A sequence that contains subsequences converging to every point in the infinite set $\{1, 1/2, 1/3, 1/4, 1/5, \dots\}$.
- (d) A sequence that contains subsequences converging to every point in the infinite set $\{1, 1/2, 1/3, 1/4, 1/5, \dots\}$, and no subsequences converging to points outside of this set.

Exercise 2.5.5. Assume (a_n) is a bounded sequence with the property that every convergent subsequence of (a_n) converges to the same limit $a \in \mathbb{R}$. Show that (a_n) must converge to a .