Homework #4
Due Monday, September 8

Exercise 2.3.1. Let $x_n \geq 0$ for all $n \in \mathbb{N}$.

(a) If $(x_n) \to 0$, show that $(\sqrt{x_n}) \to 0$.

(b) If $(x_n) \to x$, show that $(\sqrt{x_n}) \to \sqrt{x}$.

Exercise 2.3.4. Let $(a_n) \to 0$, and use the Algebraic Limit Theorem to compute each of the following limits (assuming the fractions are always defined):

(a) $\lim \left(\dfrac{1 + 2a_n}{1 + 3a_n - 4a_n^2} \right)$

(b) $\lim \left(\dfrac{(a_n + 2)^2 - 4}{a_n} \right)$

(c) $\lim \left(\dfrac{\frac{2}{a_n} + 3}{\frac{1}{a_n} + 5} \right)$

Exercise 2.3.7. Give an example of each of the following, or state that such a request is impossible by referencing the proper theorem(s):

(a) sequences (x_n) and (y_n), which both diverge, but whose sum $(x_n + y_n)$ converges;

(b) sequences (x_n) and (y_n), where (x_n) converges, (y_n) diverges, and $(x_n + y_n)$ converges;

(c) a convergent sequence (b_n) with $b_n \neq 0$ for all n such that $(1/b_n)$ diverges;

(d) an unbounded sequence (a_n) and a convergent sequence (b_n) with $(a_n - b_n)$ bounded;

(e) two sequences (a_n) and (b_n), where (a_nb_n) and (a_n) converge but (b_n) does not.

Exercise 2.3.10. Consider the following list of conjectures. Provide a short proof for those that are true and a counterexample for any that are false.

(a) If $\lim(a_n - b_n) = 0$, then $\lim a_n = \lim b_n$.

(b) If $(b_n) \to b$, then $|b_n| \to |b|$.

(c) If $(a_n) \to a$ and $(b_n - a_n) \to 0$, then $(b_n) \to a$.

(d) If $(a_n) \to 0$ and $|b_n - b| \leq a_n$ for all $n \in \mathbb{N}$, then show that $(b_n) \to b$.