

Homework #2

Due Monday, August 25

Exercise 1.3.1. (a) Write a formal definition in the style of Definition 1.3.2 for the *infimum* or *greatest lower bound* of a set.

(b) Now, state and prove a version of Lemma 1.3.8 for greatest lower bounds.

Exercise 1.3.7. Prove that if a is an upper bound for A , and if a is also an element of A , then it must be that $a = \sup A$.

Exercise 1.3.9. (a) If $\sup A < \sup B$, then show that there exists an element $b \in B$ that is an upper bound for A .

(b) Give an example to show that this is not always the case if we only assume $\sup A \leq \sup B$.

Exercise 1.4.1. Recall that \mathbb{I} stands for the set of irrational numbers.

(a) Show that if $a, b \in \mathbb{Q}$, then ab and $a + b$ are elements of \mathbb{Q} as well.

(b) Show that if $a \in \mathbb{Q}$ and $t \in \mathbb{I}$, then $a + t \in \mathbb{I}$ and $at \in \mathbb{I}$ as long as $a \neq 0$.

(c) Part (a) can be summarized by saying that \mathbb{Q} is closed under addition and multiplication. Is \mathbb{I} closed under addition and multiplication? Given two irrational numbers s and t , what can we say about $s + t$ and st ?

Exercise 1.4.3. Prove that $\bigcap_{n=1}^{\infty} (0, 1/n) = \emptyset$. Notice that this demonstrates that the intervals in the Nested Interval Property must be closed for the conclusion of the theorem to hold.