

## Homework #2

Due Monday, August 25

**Exercise 1.3.1.** (a) Write a formal definition in the style of Definition 1.3.2 for the *infimum* or *greatest lower bound* of a set.

(b) Now, state and prove a version of Lemma 1.3.8 for greatest lower bounds.

**Exercise 1.3.7.** Prove that if  $a$  is an upper bound for  $A$ , and if  $a$  is also an element of  $A$ , then it must be that  $a = \sup A$ .

**Exercise 1.3.9.** (a) If  $\sup A < \sup B$ , then show that there exists an element  $b \in B$  that is an upper bound for  $A$ .

(b) Give an example to show that this is not always the case if we only assume  $\sup A \leq \sup B$ .

**Exercise 1.4.1.** Recall that  $\mathbb{I}$  stands for the set of irrational numbers.

(a) Show that if  $a, b \in \mathbb{Q}$ , then  $ab$  and  $a + b$  are elements of  $\mathbb{Q}$  as well.

(b) Show that if  $a \in \mathbb{Q}$  and  $t \in \mathbb{I}$ , then  $a + t \in \mathbb{I}$  and  $at \in \mathbb{I}$  as long as  $a \neq 0$ .

(c) Part (a) can be summarized by saying that  $\mathbb{Q}$  is closed under addition and multiplication. Is  $\mathbb{I}$  closed under addition and multiplication? Given two irrational numbers  $s$  and  $t$ , what can we say about  $s + t$  and  $st$ ?

**Exercise 1.4.3.** Prove that  $\bigcap_{n=1}^{\infty} (0, 1/n) = \emptyset$ . Notice that this demonstrates that the intervals in the Nested Interval Property must be closed for the conclusion of the theorem to hold.