Cylindrical and Spherical Coordinates

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Outline

Cylindrical Coordinates

Cylindrical Coordinates

Cylindrical coordinates are another coordinate system on space. All you do is put polar coordinates in the xy-plane with the third coordinate being the z-coordinate in Cartesian coordinates.

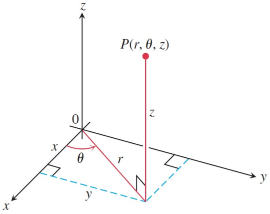


FIGURE 15.46 The cylindrical coordinates of a point in space are r, θ, and z.

Figure: Cylindrical Coordinates

Changing from Rectangular to Cylindrical Coordinates

Changing from Rectangular to Cylindrical Coordinates

The equations for changing rectangular coordinates to cylindrical coordinates should be very familiar.

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z. \end{aligned}$$

Another important conversion is

$$x^2 + y^2 = r^2.$$

Spherical Coordinates

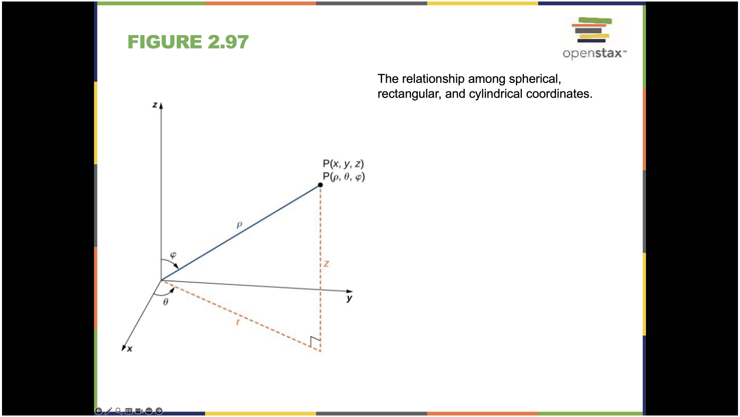
Spherical Coordinates

Spherical coordinates are another coordinate system on space. The coordinates are ordered triples (ρ, θ, φ).

The first coordinate, ρ, is the distance from the origin to the point in space. The second coordinate, θ, is the angle from the positive x-axis to the projection of the segment from the origin to the point in space into the xy-plane. The third coordinate, φ, is the angle from the positive z-axis to the segment from the origin to the point in space.

See the figure on the next slide.

Figure: Spherical Coordinates



Changing from Rectangular to Spherical Coordinates

Changing from Rectangular to Spherical Coordinates

Theorem: Converting among Spherical, Cylindrical, and Rectangular Coordinates

Rectangular coordinates (x, y, z) and spherical coordinates (ρ, θ, φ) of a point are related as follows:

$$
\begin{aligned}
x &= \rho \sin\varphi \cos\theta \\
y &= \rho \sin\varphi \sin\theta \\
z &= \rho \cos\varphi \\
\rho^2 &= x^2 + y^2 + z^2 \\
\tan\theta &= \frac{y}{x} \\
\varphi &= \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)
\end{aligned}
$$

Changing from Rectangular to Spherical Coordinates

Theorem: Converting among Spherical, Cylindrical, and Rectangular Coordinates

If a point has cylindrical coordinates (r, θ, z) then these equations define the relationship between cylindrical and spherical coordinates.

$$
\begin{aligned}
r &= \rho \sin \varphi \\
\theta &= \theta \\
z &= \rho \cos \varphi \\
\rho &= \sqrt{r^2 + z^2} \\
\theta &= \theta \\
\varphi &= \arccos \left(\frac{z}{\sqrt{r^2 + z^2}} \right)
\end{aligned}
$$