Algebraic Curves by William Fulton

The Hilbert Basis Theorem

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Outline

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The Hilbert Basis Theorem

Although we have allowed an algebraic set to be defined by any set of polynomials, in fact a finite number will always do.

Theorem

Every algebraic set is the intersection of a finite number of hypersurfaces.

Proof.

Let the algebraic set be $V(I)$ for some ideal $I \subset k[X_1, \ldots, X_n]$. It is enough to show that I is finitely generated, for if $I = (F_1, \ldots, F_r)$, then $V(I) = V(F_1) \cap \cdots \cap V(F_r)$. □

To prove this fact we need some algebra.

The Hilbert Basis Theorem

A ring is said to be **Noetherian** if every ideal in the ring is finitely generated. Fields and PID's are Noetherian rings. The theorem, therefore, is a consequence of the

Hilbert Basis Theorem

If R is a Noetherian ring, then $R[X_1, \ldots, X_n]$ is a Noetherian ring.

Proof.

$\square$