The Generalized Torelli Problem: Reconstructing a Curve and its Linear Series From its Canonical Map and Theta Geometry

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1. Introduction.

Classically Torelli's theorem states that a smooth complete algebraic curve can be reconstructed from its Jacobian and its theta divisor. More specifically, if C and C' are two smooth complete algebraic curves with Jacobians $J(C)$ and $J(C')$ and theta divisors Θ and Θ', respectively, and if $(J(C), \Theta)$ and $(J(C'), \Theta')$ are isomorphic as principally polarized abelian varieties, then C and C' are isomorphic as algebraic curves.

The generalized Torelli problem is that of reconstructing a smooth complete algebraic curve and all its linear series from information encoded in the Jacobian of the curve and the Abel-Jacobi map.

Let's begin with a bit of terminology. If $f\colon X \to Y$ is a morphism, the *multiple locus* $R \subset X$ is the closure of the set $\{x \in X \mid \text{for some } z \in X, z \neq x, \text{ and } f(z) = f(x)\}$.

If we limit ourselves to reconstructing complete pencils of linearly equivalent divisors, the result is found in [2]. Specifically, Smith and Tapia-Recillas prove the following:

THEOREM 1. *If C has a g^1_d but no g^1_{d-1} and no g^2_{d+1}, for some d with $d \leq g = \text{genus}(C)$, if $\phi\colon W_{d-1} \to \mathbb{G}(d-2, g-1)$ is the canonical map on W_{d-1}, if R is the multiple locus of ϕ, and if $B = \phi(R)$, then the restriction $\phi\colon R \to B$ is a "universal" $\phi_{g^1_d}$ for C.*

If one wishes to reconstruct complete nets of linearly equivalent divisors, the result is found in [3]:

THEOREM 2. *Let C be a non-hyperelliptic curve of genus $g \geq 5$ with a g^2_d, $5 \leq d \leq g$, such that the plane model $\Gamma \subset \mathbb{P}^2$ determined by the g^2_d is birational to C and is either smooth or it just has nodes as singularities. If $\phi\colon W_{d-2} \to \mathbb{G}(d-3, g-1)$ is the Gauss map (i.e. the canonical map) on W_{d-2}, R is the multiple locus of ϕ and $B = \phi(R)$, then the dual curve Γ^* and the linear series g^2_d can be recovered from the branch locus of the restriction of the map ϕ on R, i.e. from $\phi\colon R \to B$.*

In this paper we will solve the general case of reconstructing a smooth complete curve C and all its complete g^r_d's from the Gauss maps on the various W_d's.

2. Preliminaries.

We begin by recalling several classical results concerning the structure of the Abel map and its derivative, and their relationship to the canonical map of an algebraic curve. We then recall some results of R. Smith and Tapia-Recillas relating to the generalized Torelli problem.

Assume C is a smooth connected curve of genus g over the complex numbers. Let $C^{(d)}$ denote the d-fold symmetric product of the curve, which serves as a natural parameter space for effective divisors of degree d on the curve C. Let $J(C)$ be the Jacobian variety of C, and let $\alpha\colon C^{(d)} \to J(C)$ be the Abel map

$$\alpha(p_1, \ldots, p_d) = \left\langle \sum_{i=1}^{d} \int_{p_0}^{p_i} \omega_1, \ldots, \sum_{i=1}^{d} \int_{p_0}^{p_i} \omega_g \right\rangle,$$

defined by a basis $\{\omega_1, \ldots, \omega_g\}$ of the global holomorphic one-forms on C, and by a base point $p_0 \in C$. For $1 \le d \le g$, the image $W_d = \alpha(C^{(d)})$ is an algebraic subvariety of dimension d in $J(C)$. As illustrated by the following diagram,

$$\begin{tikzcd} C \arrow[rr, "\alpha_*"] \arrow[dr, "\alpha"'] & & \mathbb{P}^{g-1} \cong \mathbb{P}(T_0(J(C))) \\ & W_1 \arrow[ur, "\gamma"'] & \end{tikzcd}$$

the Gauss map γ on W_1, which assigns to $x \in W_1$, the tangent space $T_x(W_1)$, is equivalent to the projectivized derivative α_* of the Abel map, which in turn is just the canonical map ϕ_K of the curve.

That is, if $\alpha(p) = \left\langle \int_{p_0}^{p} \omega_1, \ldots, \int_{p_0}^{p} \omega_g \right\rangle$ then the derivative $\alpha_{*,p}$ is a linear map $\alpha_{*,p}\colon T_p(C) \to T_{\alpha(p)}(J(C)) \cong T_0(J(C))$ and the projectivized derivative α_* is given by taking its image,

$$\begin{aligned} \alpha_*(p) = \alpha_{*,p}(T_p(C)) &\in G(1, T_0(J(C))) \\ &\cong \mathbb{G}(0, \mathbb{P}T_0(J(C))) \\ &\cong \mathbb{P}(T_0(J(C))) \\ &\cong \mathbb{P}^{g-1}. \end{aligned}$$

Then since the canonical map is given by

$$\begin{aligned} \phi_K(p) = \langle \omega_1(p), \ldots, \omega_g(p) \rangle &\in \mathbb{P}(H^0(C; K)^*) \\ &\cong \mathbb{P}(T_0(J(C))), \end{aligned}$$

the fundamental theorem of calculus implies that $\phi_K(p) = \alpha_*(p)$. More generally, for $1 \le d \le g-1$ the analogous diagram commutes,

$$\begin{tikzcd} C^{(d)} \arrow[rr, dashed, "\alpha_*"] \arrow[dr, "\alpha"'] & & \mathbb{G}(d-1, \mathbb{P}(T_0(J))) \\ & W_d \arrow[ur, dashed, "\gamma"'] & \end{tikzcd}$$

and α_* is again equal to the canonical map ϕ_K on $C^{(d)}$. In this case α_* and γ are only rational maps, γ is defined on the locus of smooth points of W_d, $W_d \setminus \operatorname{sing}(W_d)$, and α_* is defined on the set $\alpha^{-1}(W_d \setminus \operatorname{sing}(W_d))$.

More precisely, if $D = \{p_1, \ldots, p_d\} \in C^{(d)}$ is such a point such that $h^0(C; D) = 1$, then ϕ_K is defined at D, $\alpha(D)$ is a non-singular point of W_d, the images of the points $\{p_1, \ldots, p_d\}$ on the canonical curve

$$\phi_K(C) = \Gamma \subset \mathbb{P}(T_0(J(C)))$$

are in general position according to the geometric Riemann-Roch Theorem, and thereby $\phi_K(D) =$ the $(d-1)$-dimensional subspace of $\mathbb{P}(T_0(J(C)))$ represented by $T_{\alpha(D)}(W_d)$, which is exactly the one spanned by the d points $\{\phi(p_1), \ldots, \phi(p_d)\}$. We remark that the "span" of these points is defined as the base locus of the set of hyperplanes $H \subseteq \mathbb{P}^{g-1}$ such that as divisors on C, $\phi_K^{-1}(H) = H \cdot C \ge \sum_1^d p_i$, and of course we say the points $\{p_1, \ldots, p_d\}$ are in general position if this base locus has dimension $d-1$. Thus with these conventions, we may write

$$T_{\alpha(D)}(W_d) = \phi_K(p_1, \ldots, p_d) = \overline{\{\phi_K(p_1), \ldots, \phi_K(p_d)\}},$$

where a bar written over a set (or a divisor) denotes its span.

3. The Fundamental Example of Smith and Tapia-Recillas.

We will now describe the simplest example, taken from [2], of the Gauss map on W_2 for a curve of genus 4, which we assume to be non-hyperelliptic. Then C is a complete intersection of a quadric and a cubic surface in $\mathbb{P}^3$ and has either one or two g^1_3's according to whether the quadric has rank 3 or 4. We will recover C and the g^1_3's from the Gauss map on W_2.

Consider the diagram:

$$\begin{tikzcd} C^{(2)} \arrow[rr, "\alpha_*"] \arrow[dr, "\alpha"'] & & \mathbb{G}(1,3) \\ & W_2 \arrow[ur, "\gamma"'] & \end{tikzcd}$$

in which, if $(p, q) \in C^{(2)}$ then $\gamma(\alpha(p, q)) = \alpha_*(p, q) = \overline{pq} =$ the secant line spanned by p and q on the canonical model of C in $\mathbb{P}^3$.

The following proposition is taken from [2]:

PROPOSITION 1. *γ is a finite map, birational onto its image, having a multiple locus in W_2 consisting of either one or two components, each birational to the curve C, and on which γ restricts to the rational maps*

$$\phi_L : C \to \mathbb{P}^1$$

defined by the one or two linear series $L = g^1_3$ on C.

We may state this result as follows:

COROLLARY 2. *A non-hyperelliptic genus 4 curve C and its g^1_3's are completely determined by restricting the canonical map ϕ of the variety $C^{(2)} \cong W_2$ to its multiple locus; i.e. the multiple locus itself is birationally equivalent to as many copies of C as there are g^1_3's, and the restriction of ϕ to these copies of C gives the corresponding rational maps $\phi_{g^1_3}$.*

4. An Example with $r = 3$: A Canonical Space Curve.

Let C be the canonical image in $\mathbb{P}^3$ of a smooth complete curve of genus four. The hyperplanes in $\mathbb{P}^3$ then cut out the canonical series on C, so that C contains a g^3_6. It is well-known that C lies on a unique quadric surface $Q \subset \mathbb{P}^3$, which has rank three or four since C is nondegenerate. If the quadric Q has rank four, then C contains two g^1_3's, whereas if the quadric Q has rank three, then C contains a unique g^1_3.

Consider the diagram:

$$\begin{tikzcd} C^{(3)} \arrow[rr, dashed, "\alpha_*"] \arrow[dr, "\alpha"'] & & \mathbb{G}(2,3) \\ & W_3 \arrow[ur, dashed, "\gamma"'] & \end{tikzcd}$$

in which, if $(p, q, r) \in C^{(3)}$ then $\gamma(\alpha(p, q, r)) = \alpha_*(p, q, r) = \overline{pqr}$ = the secant plane spanned by p, q, and r on the canonical model of C in $\mathbb{P}^3$. We remark that whereas α is defined everywhere, the Gauss map γ is defined on the smooth locus of W_3, $W_3 \setminus W^1_3$, and α_* is defined on $\gamma^{-1}(W_3 \setminus W^1_3)$. More specifically, by the geometric Riemann-Roch theorem, α_* is defined at any 3-tuple of points of C not belonging to a g^1_3.

To examine the multiple locus of α_*, we look at secant planes that meet C in at least four points. Let Π be such a plane and suppose $\#(C \cdot \Pi) = k + 3$. By the geometric Riemann-Roch theorem, $h^0(D) = k + 1$, so by Clifford's Theorem, $k = r(D) \le \frac{1}{2}\deg(D) = \frac{1}{2}(k + 3)$. It follows that $k \le 3$, so the multiple locus of α_* naturally decomposes into three components:

(1) $R_1 = \{$3-tuples of points of C whose canonical images span a two plane and which are contained in a divisor of degree 4$\}$
(2) $R_2 = \{$3-tuples of points of C whose canonical images span a two plane and which are contained in a divisor of degree 5$\}$
(3) $R_3 = \{$3-tuples of points of C whose canonical images span a two plane and which are contained in a divisor of degree 6$\}$

The structure of α_* on R_1 and R_2 is examined in [2] and [3], respectively. Let $B_3 = \alpha_*(R_3)$. Then B_3 is the set of all 6-secant 2-planes to C, and since the canonical model of an algebraic curve of genus four has degree six in $\mathbb{P}^3$, every 2-plane is 6-secant. Consequently, $B_3 = \mathbb{G}(2, 3)$. Moreover, since every 2-plane is 6-secant, any 3-tuple of points of C whose canonical images span a two plane must be contained in a divisor of degree 6, e.g. the divisor of C cut out by the plane spanned by the 3-tuple. Hence, R_1 and R_2 are contained in R_3.

5. An Example with $r = 3$: An Extremal Curve of Degree 7.

Let C to be a smooth complete curve of genus six and degree seven in $\mathbb{P}^3$. By Castelnuovo's Bound, this is a degree seven curve of maximal genus in $\mathbb{P}^3$, i.e. an extremal curve. Since C is an extremal curve, it is projectively normal, and it follows that C lies on a unique quadric $Q \subset \mathbb{P}^3$, which has rank three or four since C is nondegenerate. Moreover, the intersection of a quartic surface and this quadric is a reducible curve containing two components: a line and our curve C. Whether the rank of the quadric is three or four, the ruling(s) on Q cut out a g^1_3 and a g^1_4 on C.

Consider the diagram:

$$\begin{tikzcd}
C^{(4)} \arrow[rr, dashed, "\alpha_*"] \arrow[dr, "\alpha"'] & & \mathbb{G}(3, 5) \\
& W_4 \arrow[ur, dashed, "\gamma"'] &
\end{tikzcd}$$

in which, if $(p, q, r, s) \in C^{(4)}$ then $\gamma(\alpha(p, q, r, s)) = \alpha_*(p, q, r, s) = \overline{pqrs}$ = the secant 3-plane spanned by p, q, r, and s on $C \subset \mathbb{P}^3$. We remark that whereas α is defined everywhere, the Gauss map γ is defined on the smooth locus of W_4, $W_4 \setminus W^1_4$, and α_* is defined on $\gamma^{-1}(W_4 \setminus W^1_4)$. More specifically, by the geometric Riemann-Roch theorem, α_* is defined at any 4-tuple of points of C not belonging to the g^1_4.

To examine the multiple locus of α_*, we look at secant 3-planes that meet Γ, the canonical image of C, in at least five points. Let Π be such a 3-plane and suppose $\#(\Gamma \cdot \Pi) = k + 4$. By the geometric Riemann-Roch theorem, $h^0(D) = k + 1$, so by Clifford's Theorem, $k = r(D) \leq \frac{1}{2}\deg(D) = \frac{1}{2}(k + 4)$. It follows that $k \leq 4$ and if C is non-hyperelliptic, $k < 4$. So, for a non-hyperelliptic C, the multiple locus of ϕ naturally decomposes into three components:

(1) $R_5 = \{$4-tuples of points of C whose canonical images span a 3-plane and which are contained in a divisor of degree 5$\}$
(2) $R_6 = \{$4-tuples of points of C whose canonical images span a 3-plane and which are contained in a divisor of degree 6$\}$
(3) $R_7 = \{$4-tuples of points of C whose canonical images span a 3-plane and which are contained in a divisor of degree 7$\}$

The structure of α_* on R_5 and R_6 is examined in [2] and [3], respectively. Let $B_7 = \alpha_*(R_7)$. Then B_7 is the set of all 7-secant 3-planes to Γ.

6. General Linear Series: g^r_d's in Arbitrary Genus.

Let C be a smooth complete algebraic curve. We assume initially that C is non-hyperelliptic, and so by Clifford's Theorem we must have $r < \frac{1}{2}d$.

Consider the commutative diagram

$$\begin{tikzcd} C^{(d-r)} \arrow[rr, dashed, "\phi"] \arrow[dr, "\alpha"'] & & \mathbb{G}(d-r-1, \mathbb{P}(T_0(J))) \\ & W_{d-r} \arrow[ur, dashed, "\gamma"'] & \end{tikzcd}$$

where ϕ is the canonical map, α is the Abel-Jacobi map, and γ is the Gauss map. We note that the maps ϕ and γ are only rational maps in general and will be regular exactly if C does not contain a g^1_{d-r}. In this case, W_{d-r} is smooth and W_{d-r} and $C^{(d-r)}$ are biregular, and we first assume this is the case. Thus, we assume W_{d-r} is smooth and the Abel-Jacobi map $\alpha\colon C^{(d-r)} \to W_{d-r}$ is an isomorphism. In this way we may identify $C^{(d-r)}$ with W_{d-r}.

Let R be the closure of the set

$$\{D \in C^{(d-r)} \mid \phi(D) = \phi(D') \text{ for some } D' \in C^{(d-r)},\ D \neq D'\}, \tag{1}$$

and define $B = \alpha(R)$. As motivation, $D \in C^{(d-r)}$, then by the geometric Riemann-Roch theorem, $\overline{\phi(D)}$ is a $(d-r-1)$-plane which is at least $d-r$ secant to the canonical curve. If D is in the set (1), there must be two distinct divisors D and D' in $C^{(d-r)}$ whose images on the canonical curve span the same $(d-r-1)$-plane. In particular, this plane must be at least $d-r+1$ secant to the canonical curve. So we will consider $(d-r-1)$-planes in $\mathbb{P}H^0(C,K)^* \cong \mathbb{P}^{g-1}$ which are at least $d-r+1$ secant to the canonical curve.

Let $k \geq 1$ and suppose some $d-r-1$-plane Π is $d-r+k$ secant to the canonical curve. Let E be the divisor consisting of these $d-r+k$ points. By the geometric Riemann-Roch theorem, E is an element of a g^k_{d-r+k}, and by Clifford's theorem, we must have $k \leq d-r-1$.

LEMMA 3. *With the hypothesis of this section,*

$$\begin{aligned} R &= \{D \in W_{d-r} : \#(\overline{D}\cdot\Gamma) \geq d-r+1\} \\ &= \{D \in W_{d-r} : \text{ for some } p_1,\ldots,p_k \in C, |D+p_1+\cdots+p_k| = \text{ a } g^k_{d-r+k}\} \end{aligned}$$

for some $1 \leq k \leq d-r-1$.

PROOF:

Suppose that $\#(\overline{D}\cdot\Gamma) \geq d-r+1$. Then $E = (\overline{D}\cdot\Gamma)$ is a divisor containing D and at least $d-r+k$ points for some $k \geq 1$. Hence, by the geometric Riemann-Roch theorem, E is an element of a g^k_{d-r+k}. The reverse inclusion involves a similar argument.

The condition

$$\#(\overline{D}\cdot\Gamma) \geq d-r+1$$

defines a closed subset $S \subset W_{d-r}$. Now we must show that $S = R$.

Since S is closed, to show that $R \subset S$ it suffices to show that if D and E are two divisors with $D \neq E$ and $\overline{D} = \overline{E}$, then $\#(\overline{D} \cdot \Gamma) \geq d - r + 1$. This is easy. We have $\overline{D + E} = \overline{D} = \overline{E}$ and it follows from the geometric Riemann-Roch theorem that $(D + E) \in g^k_{d-r+k}$ for some $k \geq 1$.

To show $S \subset R$, assume $D \in W_{d-r}$ and $\#(\overline{D} \cdot \Gamma) \geq d - r + 1$. By assumption, C does not contain a g^1_{d-r}, so by the geometric Riemann-Roch theorem, a general divisor E of containing $d - r$ points and contained in an element of the linear system $|(\overline{D} \cdot \Gamma)|$ gives a point of R. Since D is in the closure of these points, $D \in R$. The lemma is proved. ■

For each k, $1 \leq k \leq d - r - 1$, let

$$R_k = \{D \in W_{d-r} \colon \#(\overline{D} \cdot \Gamma) = d - r + k\}$$

and let $B_k = \alpha_*(R_k)$.

COROLLARY 3.

(1) *For each k, $1 \leq k \leq d - r - 1$,*

$$\begin{aligned} B_k = \alpha_*(R_k) &= \{\text{the set of } (d - r + k)\text{-secant } (d - r - 1)\text{-planes in } \mathbb{P}^{g-1}\} \\ &= \{\text{linear spaces of the form } \overline{D} \text{ for some } D \text{ in some } g^k_{d-r+k}\}. \end{aligned}$$

for some $1 \leq k \leq d - r - 1$.

(2) *For each k, $1 \leq k \leq d-r-1$, let $B_k = \{$the set of $(d-r+k)$-secant $(d-r-1)$-planes in $\mathbb{P}^{g-1}\}$, so that $B = \cup_k B_k$. If α now denotes the Abel map on $C^{(d)}$, $\alpha \colon C^{(d)} \to W_d$, and if $W_d \supset W^r_d = \{$ the set of g^r_d's on $C\}$, then the projectivized derivative of α gives a holomorphic bijection (thus a homeomorphism in the complex topology)*

$$\alpha_* \colon \alpha^{-1}(W^r_d) \to B_r$$

where $\alpha_*(D) = \overline{D}$.

LEMMA 4. *Using the same notation as in the last corollary, B_r is fibered in precisely one way by a family of disjoint analytic r-folds homeomorphic to $\mathbb{P}^r$; precisely, the only such r-folds on B_r are those of the form:*

$$\{\overline{D} \ : \ \text{for all } D \text{ in some fixed } g^r_d \text{ on } C\}.$$

PROOF: It is sufficient of part (2) of the previous corollary to prove the same statement for the set

$$\alpha^{-1}(W^r_d) = \{\text{all divisors } D \text{ on } C \text{ such that } |D| \text{ is a } g^r_d\}.$$

Let $X \subseteq C^{(d)}$ be any analytic set homeomorphic to $\mathbb{P}^r$. Then Abel's map, restricted to X, factors through the universal covering space of $J(C)$:

$$\begin{tikzcd} & \mathbb{C}^g \arrow[d, "\pi"] \\ X \arrow[ur, "\tilde{\alpha}"] \arrow[r, "\alpha"] & W_d \subset J(C) \end{tikzcd}$$

Since X is compact and $\tilde{\alpha}$ is analytic, the maximum modulus principle implies that $\tilde{\alpha}(X)$ is a single point, so that $\alpha(X)$ is also a single point. By the converse of Abel's theorem, $X = |D| =$ the set of divisors belonging to some g^r_d on C. ■

Now we state a theorem describing the Gauss map on W_{d-r} for a generic d-gonal curve.

THEOREM 5. *If C has a g^r_d but no g^1_{d-r} and no g^{r+1}_{d+1}, for some d with $d \leq g =$ genus(C), if $\phi\colon W_{d-r} \to \mathbb{G}(d-r-1, g-1)$ is the canonical map on W_{d-r}, if R is the multiple locus of ϕ, $B = \phi(R)$, then R naturally decomposes into subsets $R_1, \dots, R_{d-r-1}$, where R_j is the closure of the set of divisors D in C contained in a divisor E belonging to a g^j_{d-r+j}, and B naturally decomposes into subsets $B_1, \dots, B_{d-r-1}$, where B_j is the set of $(d-r+j)$-secant $(d-r-1)$-planes. Further, the restriction, $\phi\colon R_r \to B_r$ is a "universal" $\phi_{g^r_d}$ for C. Precisely, B_r is uniquely a union of a family of disjoint algebraic curves homeomorphic to $\mathbb{P}^r$; these are precisely the curves of the form $\alpha_*(|g^r_d|)$ where $|g^r_d| \subset C^{(d)}$ is a linear system of linear divisors of degree d and dimension r; for any one of these curves $X_L \subset B_r$, $L = g^r_d$, the inverse image $\phi^{-1}(X_L) \subset R_r$ is an algebraic curve bijectively birational to C; after normalizing, the Gauss map of W_{d-r}, restricted to $\phi^{-1}(X_L)$, induces the rational map ϕ_L determined by the linear series $L = g^r_d$, i.e. the following diagram commutes:*

$$\begin{tikzcd} C \arrow[r, "\phi_L"] \arrow[d, "\nu"] & \mathbb{P}^r \arrow[d, "\nu"] \\ \phi^{-1}(X_L) \arrow[r, "\phi"] & X_L \end{tikzcd}$$

in which ν denotes normalization.

PROOF: We have proved all the but the last two statements. Since $\phi\colon R_r \to B_r$ is a proper map with finite fibers it is a finite map, $\phi^{-1}(X_L)$ is an r-fold, and the map

$$\begin{aligned} f: C &\to \phi^(X_L) \\ p &\mapsto (\overline{D(p)} \cdot C) \setminus \{p\} \end{aligned}$$

is a holomorphic bijection, hence it is the normalization map, where $D(p) \in g^1_d = L$ is the unique divisor of L containing p. The normalization diagram

$$\begin{tikzcd} C \arrow[r, "\phi_L"] \arrow[d, "f = \nu"'] & {|L| \subset C^{(d)}} \arrow[d, "\alpha_* = \nu"'] \\ \phi^{-1}(X_L) \arrow[r, "\phi"] & X_L \subset B_r \end{tikzcd}$$

is now seen to commute.

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