Basic Algebraic Geometry, Volume 2
Chapter 5, Section 2: Sheaves

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1. Presheaves

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Definition. Let X be a given topological space. Suppose that with every open set $U \subset X$ we have associated a set $\mathcal{F}(U)$ and with any open sets $U \subset V$ a map

$$\rho_U^V : \mathcal{F}(V \to \mathcal{F}(U).$$

This sytem of sets and maps is a **presheaf** if the following conditions hold:

(1) If U is empty, the set $\mathcal{F}(U)$ consists of 1 element;

(2) ρ_U^U is the identity map for any open set U;

(3) for any open sets $U \subset V \subset W$, we have

$$\rho_U^W = \rho_U^V \circ \rho_V^W.$$

(2) A presheaf is sometimes denoted $\mathcal{F}$. If we need to emphasize the that the maps ρ_U^V refer to $\mathcal{F}$, we denote them by $\rho_{U,\mathcal{F}}^V$.

(3) If all the sets $\mathcal{F}(U)$ are groups, rings, or modules over a ring A, and the maps ρ_U^V are homomorphisms of these structures, then $\mathcal{F}$ is a presheaf of groups, rings, or A-modules.

(4) The choice $\mathcal{F}(\emptyset)$ is irrelevant. For a sheaf of groups, we let $\mathcal{F}(\emptyset)$ be the group with one element.

(5) If $\mathcal{F}$ is a presheaf on X and $U \subset X$ is an open set, then sending V to $\mathcal{F}(V)$ for open subset $V \subset U$ obviously defines a presheaf on U. This is the **restriction** of the presheaf $\mathcal{F}$ and is denoted $\mathcal{F}|_U$.

Start with Example 1 on page 29 in the box.

2. The Structure Presheaf

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3. Sheaves

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4. Stalks of a Sheaf

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